

What math should we teach,
now that cell phones can do it all?

Warren Esty
Professor of Mathematics, Emeritus
Montana State University
wwesty@gmail.com

(The second last slide was inserted Sept. 14, 2018 to provide relevant links.)

The world is changing rapidly.

Is teaching keeping up?

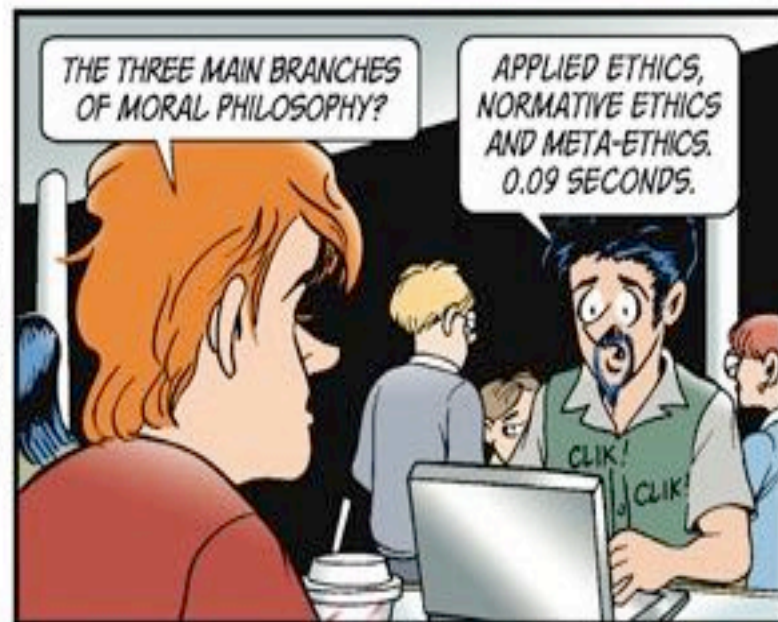
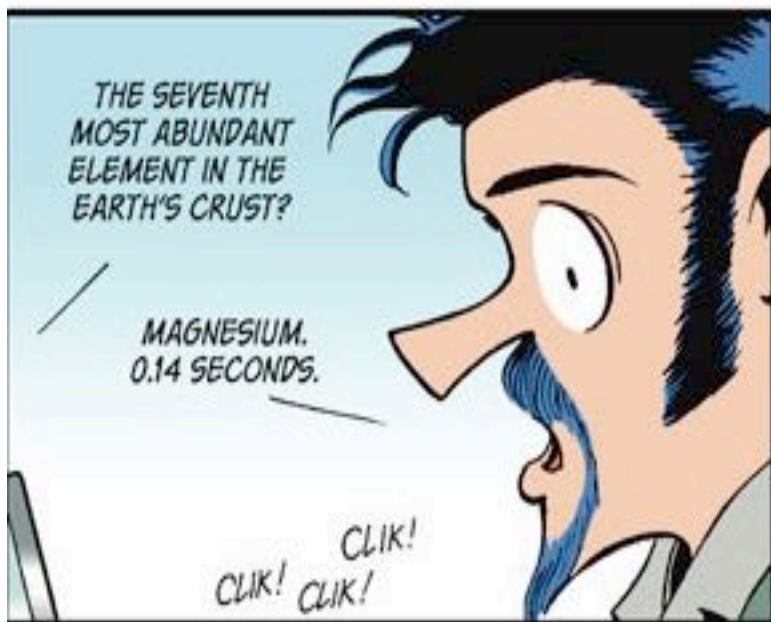
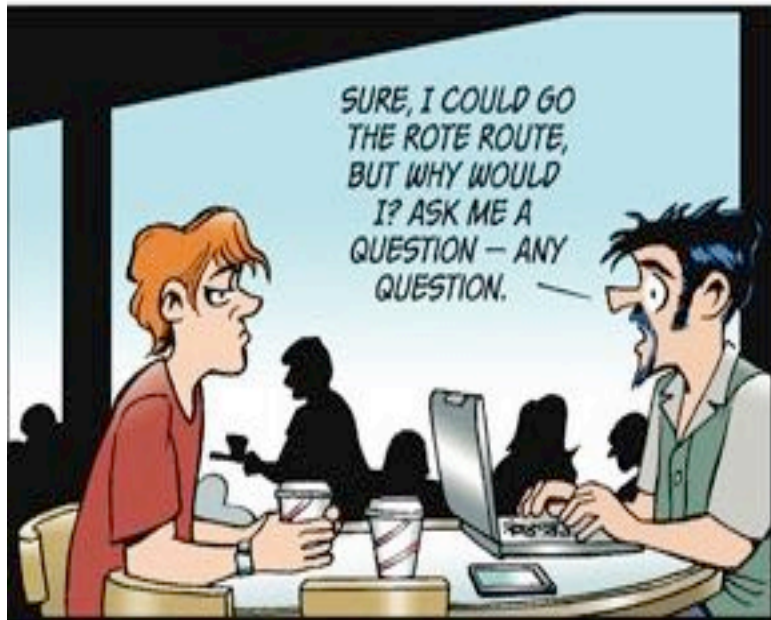
What do we teach?

Information

Skills

Impart information, computational skills

- Information is cheap
- Facts are on the web





Computational skills

- Computational skills are cheap
- If anyone can compute it, calculators and computers can do it accurately and faster
- Cell phones can do it
- iPads can do it

The value of math skills

I can buy

- a great calculator for \$100
- I can use Maple, Mathematica,
- Wolfram Alpha, etc
- Apps that **SHOW THE STEPS** for a few bucks

Expert Systems

- “Expert systems” are replacing people
- Read x-rays better than radiologists
- Diagnose more accurately than doctors
- And, something similar in every other profession

You should be afraid,
very afraid

if all you know of math is

- 1) facts, and
- 2) computational skills

Theorem: The value of traditional mathematical skills has gone way down.

Corollary: We should refocus our teaching toward skills that **add value**.

What adds value?

What adds value?

- Knowing how to use calculators, computers, and similar apps
- Knowing when to use various math algorithms
- Experience with problems

- Also, of course, many traditional activities (especially developing concepts)

Add value by

- Preparing for lifelong learning
Learn to read!
- Learning to reason logically

The problem with searches

- “I can just look it up.”
- However, “it” must come to mind
- And you must be able grasp it when you find it

Модернизация доберется до нефтяных скважин

14.12.2010



Запасы нефти в России исчерпаны уже на 50%, а уровень износа мощностей в нефтепереработке достигает 80%, сообщается в материалах, подготовленных к заседанию Совета безопасности РФ. В ходе заседания президент Дмитрий Медведев поручил правительству разработать доктрину энергетической безопасности и определить неотложные

меры по ее реализации.

На данный момент РФ занимает седьмое место в мире по доказанным запасам нефти — более 74 млрд баррелей. прогнозные ресурсы газа оцениваются в 165 трлн кубометров. Неизвестно, какие именно данные взяты за точку отсчета при указании исчерпания половины нефтезапасов страны. Вообще, максимально приближенные к реальности цифры такого рода относятся к категории стратегически важной информации, а поэтому не озвучиваются.

Читайте также: ["ВР устроил распродажу мелочевки"](#)

- Learning to make (the right) things
“come to mind”

Solve

$$\sin^2 x + \sin x = 0.80$$

Graph

$$x^2 + 3xy + y^2 + 6y = 100$$

Reading

- Reading, with full comprehension, helps bring things to mind

We found that kids hit a wall, and that wall is called fourth grade. At that moment, a kid shifts from learning to read to **having to read in order to learn.**

-- David Britt, Children's Television Workshop President, in 1992

Half of all kids
never make that transition.

-- Colette Daiute, Harvard Professor
of Human Development, in 1992

Imagine how much worse
the statistics would be
if they were about
the fraction of kids who

can read *mathematics*
to learn *mathematics*.

Why is reading important?

Do you really think students can read math?

- Why should they be able to?
- Who ever taught them to?
- Who ever **required** them to?

Math is difficult to read

- It is concise and precise
in a non-concise and non-precise age
- $[0, 1]$
- $(0, 1)$
- $\{0, 1\}$
are different in important ways
that are alien to our students

43 College calc profs asked their students

- 8% read most of the chapters
- 17% read sections they didn't understand from lectures
- **69%** typically started by working homework and turned to examples if they had trouble
- 3% said they never opened the book.

Is Math a Language?

- Communication
- By symbols
- Non-instinctive
- Conventional, learned meanings
- Shared by a community

Algebraic Language has

- Vocabulary nouns, pronouns, verbs,
 “expression”, “factor”, ...
- Grammar $3(x + 1)$, $3x + 1$
- Syntax $2x^2$ is not $(2x)^2$
- Pronunciation $\{x \mid x^2 > 25\}$
- Synonyms If $x > 5$, then $x^2 > 25$,
 For all $x > 5$, $x^2 > 25$.
- Negations negate: “If $x^2 > 25$, then $x > 5$.”
- Conventions $3x^2$
- Abbreviations
- Sentence and paragraph structure

Placeholders

- $3(x + 4) = 18$ (x is **not** a placeholder here — it is a “free variable” or an “unknown”)
- $3(x + 4) = 3x + 12$ (x **is** a placeholder here)
- $3(c + 4) = 3c + 12$ (c is in the place of x)

- Let $f(x) = x^2$. x is a placeholder.
“ f ” is not a number, “ $f(x)$ ” is.
- Find $f(x+h) =$

How do you add fractions?

- Explain this in English

Explain this in Mathematics

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

[For this talk, many problems have been omitted.]

Precalculus prerequisites. Quiz

All these should ring a bell. Fill in the result, if possible, and leave it blank if not.

- $(x + a)^2 =$
- $(x + a)(x + b) =$
- $x^2 - 9 =$
- $x^2 + 9 =$

What is the best way
to learn a language?

Spanish?

German?

Computers?

The best way to learn a Language?

- Start very young
- Use it a lot
- Interact with others who use it

Our children can't "Start young"

- Many elementary-school teachers don't know the language
- And avoid it
- And the curriculum lets them
- We often start the language in 6th or 7th grade or later.

Few EI-Ed students choose extra math

- They don't have time in their curriculum
- They are not expected to be responsible for algebra
- A math-as-a-language course is not traditional
- Few colleges have one

Linguists assert: It is difficult to have and retain thoughts without the proper language in which to categorize and express them.

- E.g. Physicians need medical terms
- Musical terms (e.g. chords)
- Symbolic mathematics

The Quadratic Theorem

- If $ax^2 + bx + c = 0$ and a is not 0,
- Then $x = \dots$
- Find x when $2x^2 - kx = 12$
- Find y when $x^2 + 3x + 5y^2 - 12y = 100$
- Find b when $c^2 = a^2 + b^2 - 2ab \cos(C)$
- Find x when $\sin x + (\sin x)^2 = 0.82$

The Language of Mathematics

- 1. Algebra as a language
Abstraction, Patterns, Order, Reading, Arithmetic methods expressed
- 2. Sets, functions, algebra
Notation, Methods expressed
- 3. Logic for Mathematics (logical equivalences)
- 4. Sentences, Variables, Generalizations, Existence Statements, Negations
- 5. Proofs (paragraphs in the language)

A course that adds value

- Who will go to bat for one?
- It is not a traditional course
- Previous teachers, administrators, parents, didn't take this course (It didn't exist)
- Math profs usually don't care much about elementary ed, or have much influence over it
- Not everyone realizes the language aspects of mathematical symbolism – shouldn't students just get that by osmosis in their math classes?
- Colleges readily accept new courses if students will take them, but
- Who will take it, if it is not required?

A text for a language course

- *The Language of Mathematics*, by Esty
- Website here: estymath.com
- Chapter 1.1 on Academia.edu under “Warren Esty”
- Relevant articles on Academia.edu under “Warren Esty”
- Contact me at wwesty@gmail.com

Conclusion: The world has changed

- Information is incredibly cheap
- Calculations are incredibly cheap
- Theorem: **Much of the math we have been teaching is not worth much.**
- Learning to read is not easy
- Learning to read is worth a lot
- We must enable our teachers to help students learn to read *Mathematics*.