Final Exam on *Proof* Name _________________________

Chapters 1-5.2. Fall 2013

1. (6 pts) Give

a) $\{2, 4\}^c$

b) The power set of $\{a, b\}$

2. (5, 5, 3 pts) Give the sentence-form definitions of

a) rational number

[for parts (b) and (*c*), The domains and codomains are as in the text: $f: A \to B$ and $g: B \to C$.}

b) one-to-one

c) g is onto C

3. (15 pts) Give the negation (in positive form) of these. Translate first if appropriate.

a) $S \subset T$

b) S is bounded

c) [f and c are given] For $\varepsilon > 0$ there exists $\delta > 0$ such that $|f(x) - f(c)| < \varepsilon$ whenever $|x - c| < \delta$.

4. (12 pts) Give the contrapositive of this. For part (b) restate it as a conditional first.

a) If, for each $\varepsilon > 0$ there exists x in S such that $x > b - \varepsilon$, then $\sup(S) \geq b$.

b) No number less than b is in S.

5. (10 pts) Let $x\#y = 2x$ if $x \geq y$ and $x\#y = 3y$ if $x < y$.
Solve for x: $x\#40 = (2\#4)(5\#3)$. [The parentheses on the right denote multiplication.]

6. (12 pts) Make a truth table to determine if "A => not B "is logically equivalent to "not B or not A."
(Include every appropriate column)

7. (16 pts) True or false (No reason required]

a) T F If $|x| < y$, then $x < y$.

b) T F If $|x| < c$, then $x^2 < c^2$.

c) T F If $xy > 25$ then $x > 5$ or $y > 5$.

d) T F $x + |y| \leq |x + y|$.

8. (12 pts) Suppose this is true: (*) If $x > 5$, then $f(x) \leq 3$ or $f(x) > 10$.

a) What can be deduced from (*) and $x = 6$ and $f(x) > 4$?

b) What can be deduced from (*) and $f(x) = 6$?

c) What can be deduced from (*) and $x^2 = 36$ and $f(x) = 7$?

9. (8 pts) State the logical equivalence we call

a) Cases

b) A Hypothesis in the Conclusion

Proofs and Conjectures [12 pts each. Do them on blank paper provided. Cite reasons, of course.]

10. Prove by induction: $1 + 2 + 4 + 8 + ... + 2^n = 2^{n+1} - 1$.

11. Prove, from the definition of supremum as least upper bound (and not using subsequent theorems): Theorem: If $R \subset S$, then $\sup(R) \leq \sup(S)$.

---- [For functions, the domains and codomains are as in the text: f: $A \to B$ and g: $B \to C$.]

12. Conjecture: If $g \circ f$ is onto C, then g is onto C.

13. Prove: If $R \subset S$, then $f^{-1}(R) \subset f^{-1}(S)$.

14. Conjecture: $x \geq b - c$ for all $c > 0 \Rightarrow x \geq b$. [Note the new "≥" instead of ">" in the hypothesis.]

15. Definition: p is a **limes point** of S iff $p \in S$ and for each $\delta > 0$, $(p - \delta, p + \delta) \cap S^c$ is not empty.
Conjecture: If p is a limes point of S and $p \in T$, then p is a limes point of $S \cap T$.

— Do TWO (2) [your choice] of the remaining three.

Option 16. Given the definition of "$a < b$" (Definition 3.1.3) and its prior results, prove
Theorem: $a < b$ and $c > 0 \Rightarrow ca < cb$. [Cite reasons and do not skip steps.]

Option 17. Conjecture: If f is onto B and $g \circ f$ is one-to-one, then g is one-to-one.

Option 18. Prove: If $S \subset f^{-1}(T)$, then $f(S) \subset T$.