Final Exam on *Proof*. Name:_____
Sections 1.1-5.2. Spring 2014 initial each sheet of extra paper

1. (4 pts) Write out a good pronunciation of "$\{x \mid x^2 \in S\}$".

2. (6 pts) State the logical equivalences we know by these names:
a) A Hypothesis in the Conclusion

b) Cases

3. (15 pts) True or false? [No reason required.]

a) T F $|x| < |y| \Rightarrow x^2 < y^2$

b) T F If $|x - c| < d$, then $|x| < d + c$.

c) T F To prove "$(A \Rightarrow B) \Rightarrow (C \Rightarrow D)$," C can be regarded as a hypothesis.

d) T F If $A \Rightarrow$ not B, then $B \Rightarrow$ not A.

e) T F If $x \in (1, 6]$ there is $y \in (1, 6]$ such that $y > x$.

4. (6 pts) These might have grammatical mistakes or unconventional usages. Which ones? Give a short and specific indication of the error.
a) $S \cup T$ iff $x \in S$ or $x \in T$. b) Let $y \in f^{-1}(T)$ c) Let $n \in \{1, 2, 3, ..., \infty\}$

5. (15 pts) Give the negation (in positive form) of these. [You may need to translate first.]
a) [f is given] $x > 3 \Rightarrow f(x) > 12$

b) [$\{a_n\}$ is given.] For any $m > 0$ there exists n^* such that $a_n > m$ if $n > n^*$.

c) [S is given] S is bounded above.

6. (10 pts) Let $n^* = n/2$ if n is even and $n^* = 5n$ if n is odd. Solve for n: $n^* = (14^*)(3^*)$.

7. (8 pts) Give the sentence-form definition of
a) (set) *union*

b) *onto*

8. (24 pts) Suppose this is given: If $f(x) \leq 8$, then $|x| < 3$.
Determine which of these follow logically.

a) FL not FL If $f(x) < 6$, then $x \leq 3$.

b) FL not FL If $|f(x)| < 4$, then $|x| < 4$.

c) FL not FL If $f(x) < 7$, then $|x| \leq 2$.

d) FL not FL If $x > 6$, then $|f(x)| > 7$.

e) FL not FL $f(x) > 6$ or $x \neq 3$.

f) FL not FL If $x > 5$ or $x < -4$, then $f(x) > 8$.

9. (12 pts) Assume this sentence is true: "If M is frid and not taj, then it is whompy." What can you deduce, if anything, from that and the following additional fact?

a) M is not taj and not whompy.

b) M is frid and whompy.

c) M is not whompy.

10. (12 pts) Make a truth table, with **all** relevant columns, to determine if "A => not B" is logically equivalent to "not(A => B)." [As always, put the column for A on the left.] Be sure to conclude if they are or are not logically equivalent.

*** C**ontrapositive**

11. (10 pts) Give the contrapositive of these [You may need to translate first]

a) b is an upper bound of S

b) If $x > c - \varepsilon$ for all $\varepsilon > 0$, then $x \geq c$.

12. (18 pts) Examples:

a) $f(S) \cap f(T) = f(S \cap T)$ is false as a generalization. Give an example that proves it false.

b) $x - a < b \Rightarrow |x| < a + b$ is false as a generalization. Give an example that proves it false.

c) Definition: f is increasing iff $x < z \Rightarrow f(x) \leq f(z)$.
Prove "$f(x) = x^2$ is increasing" is false.

^^^^^^^^^^^^^^^ **Proofs and Resolutions**

Instructions:

- Do proofs from the **definitions** (not later theorems)
- **Do not cite very similar results** to "prove" things.
- **Cite reasons** for each step.
- Counterexamples must be complete.

(12 pts each) Use the blank paper provided. Initial each sheet you use.

13. Prove by induction: If $a_1 = 2$ and $a_n = a_{n-1} + 2n$ for $n \geq 2$, then $a_n = n(n+1)$.

14. Resolve this Conjecture: If x is irrational and y is rational, then $x - y$ is irrational.

15. Prove: For $m > 0$ there exists $h > 0$ such that $1/(2x) > m$ if $0 < x < h$.

16. Resolve this Conjecture: If $g \circ f$ is one-to-one, then f is one-to-one.

Do ONE of the next two options [Be clear which one you choose to do]:

Option 17A: Prove: $f^{-1}(S \cap T) \subset f^{-1}(S) \cap f^{-1}(T)$

Option 17B: Prove: $f(S \cap T) \subset f(S) \cap f(T)$