Proof, Spring 2013. Name ____________________________
Proof, Chapter 4, sets. initial each sheet you use

1. (5 pts) Define *bound* in sentence-form.

2. (25 pts) True or false, no reason required.
a) T F If $R \subset S \cup T$, then $R \subset S$ or $R \subset T$.

b) T F $\sup\{f^2(x)\} \leq (\sup\{f(x)\})^2$.

c) T F $S \cap T \in \boldsymbol{P}(T)$. [The notation $\boldsymbol{P}$ is for the power set]

d) T F If $S \cup T = S$, then $S \subset T$.

e) T F If there exists x in S such that $x > k$, then $\sup(S) > k$.

Resolve conjectures with a proof or disproof. (First say if they are true or false.)
Prove the true ones **using the *definitions* of the terms as prior results**.
– Do not use or cite similar theorems in the text.
– If it was done in the text, class, or the homework, do it again here.
Cite reasons
– Steps at the level of the course without cited reasons will cause the problem to get less than full credit.

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3. (12 pts) Prove this theorem. (There is a very similar theorem in the text. Do NOT use the similar theorem to prove this. Use the definition of sup as least upper bound.)
Theorem: If $M = \sup S$ and $S \neq \emptyset$ and $\varepsilon > 0$, then there is x in S such that $x > M - \varepsilon$.

4. (10 pts) Prove [assuming the sups are finite]: If $S \subset T$, then $\sup(S) \leq \sup(T)$.
[Do not cite a similar result to prove this. Prove it using the definitions.]

***** Do FOUR (4) of the remaining 5. Omit one. (12 points each)

5. Conjecture: If $S \subset T$ and b is an upper bound of T, then b is an upper bound of S.

6. Conjecture: If $a > b$ there exists a natural number n such that $a - 1/n > b$.

[Omit one of 6-10.]

7. Conjecture: $\boldsymbol{P}(S \cap T) \subset \boldsymbol{P}(S)$.

8. Conjecture: If $c > b - \varepsilon$ for all $\varepsilon > 0$, then $c \geq b$.

9. Conjecture: If $\sup S$ is finite and $\sup S$ is not in S, then S has at least two elements.