Methods of Proof. Exam on Chapter 3 of *Proof.* Name:___________________________
Spring 2014 initial each sheet of extra paper

1. (15 pts) True or false? [No reason required here, but see #2.]
a) T F If (A and B) $\Rightarrow C$ is true, then $A \Rightarrow C$ is true.
b) T F If $A \Rightarrow B$ is true, then $A \Rightarrow$ (B or C) is true.
c) T F If $|x| < |y|$ and $|a| < |b|$, then $|x| - |b| < |y| - |a|$.
d) T F If $|x| < y$, then $|x + c| < |y| + |c|$.
e) T F If $|x - c| < 1$, then $|x^2 - c^2| < 1$.

2. (4 pts) At least two of the above are false. Select two (be clear which two) and give specific counterexamples.

3. (15 pts) Suppose f is a given function and this is given: "If $x > 4$, then $f(x) \leq -5$ or $f(x) > 10$." Which of these follow logically?
a) FL not FL $f(5) \neq 7$.

b) FL not FL If $x > 4$ and $f(x) > -5$, then $f(x) > 9$.

c) FL not FL If $f(x) = 7$, then $x < 4$.

d) FL not FL If $|f(x)| \leq 3$, then $x < 5$.

e) FL not FL $x \leq 4$ or $|f(x)| \geq 5$.

4. (16 pts) Give the negation, in positive form, of

a) $S \subset T$.

b) If a and b are greater than 0, then $ax^3 + bx^2 + c = 0$ has at most one solution.

c) [f is given.] For each $\varepsilon > 0$ there exists c such that $x > c$ implies $|f(x)| < \varepsilon$.

d) [S is given.] There exists x in S such that for all y in S, $x \leq y$.

[over]

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Demonstrate that you know how proofs and disproofs are written. If you did it on the homework, or it was done in class or in the text, **do it again here**.
Resolve conjectures with proofs or disproofs. Do not cite results to prove themselves, make sure your steps are prior. **Cite justifications!** (of work at the level of the problem–but not for algebra.) Do work on separate sheets provided.

Correct work without reasons for the steps (unless they are much lower level) **will not get full credit**.

(10 points each). **Do work on separate sheets provided**.

5. [For this one, nothing about rational numbers is prior except the definition of "rational number."]
Prove: If x is rational and $x - y$ is irrational, then y is irrational.

6. Prove the theorem by induction for all $n \geq 1$.
Theorem: $1(2) + 2(3) + 3(4) + \ldots + n(n+1) = n(n+1)(n+2) / 3$.

7. [For this problem, the domain of f is the reals.]
Definition: f is strictly increasing iff $x < z \Rightarrow f(x) < f(z)$.
Conjecture: If f is strictly increasing, then, for each c, $f(x) = c$ has at most one solution.

8. Let $f(x) = f(x) = \dfrac{1}{\sqrt{x}}$.
Prove: For $\varepsilon > 0$ there exists b such that if $x > b$, then $f(x) < \varepsilon$.

9. [Inequalities from Section 3.1.] Prove this using only Definition 3 ("$a < b$ iff $b - a > 0$") and prior results which include the axioms, but use nothing after Definition 3:
Theorem: If $c < 0$ and $a < b$, then $ca > cb$.