Exam on Chapters 1 and 2 of *Proof.* Name:______ Spring 2014

1. (10 pts) Suppose this is true: If $x \le 3$, then f(x) > 12. Which of the following follow logically (FL)?

a) FL not FL If x < 3, then f(x) > 10.

b) FL not FL If $f(x) \le 10$, then x > 3.

c) FL not FL f(x) > 11 when x > 4.

- d) FL not FL If x = 1, then f(x) > 13.
- e) FL not FL $x \leq 3$ or $f(x) \leq 12$.

2. (10 pts) Suppose this is a fact: If f(x, y) < 8, then x < 2 or y > 10. What can be deduced from that and this additional fact?

- a) f(x, y) < 7 and x > 3.
- b) f(x, y) > 8 and x < 2.
- c) f(x, y) < 9 and x > 2.
- d) f(3, y) = 7

e) y < 8 and x > 5

- 3. (6 pts) State the Logical Equivalence we call
- a) A Hypothesis in the Conclusion

b) Cases

- 4. (6 pts) Let S = [1, 5).
- a) T F For all x in S there exists y in S such that y > x.
- b) T F For all x in S there exists y in S such that y < x.
- c) T F There exists y in S such that for all x in S, $y \ge x$.

- 5. (18 pts) Give the negation (in positive form) of
- a) [Let *T* and *b* be given.] If $x \in T$, then $x \le b$.

b) At least one pile has at least 7 chips

c) $ax^4 + bx^3 + c = d$ has 4 solutions if a > 0 and b < 0.

- d) If $x > k \varepsilon$ for all $\varepsilon > 0$, then x > k.
- e) [Let *f* be given.] For any *m* there exists n^* such that if $n > n^*$ then f(n) > m.
- f) *S* is bounded above. [Translate first, then negate.]
- 6. (6 pts) Solve for a in " $c^2 = a^2 + b^2 2ab \cos C$."

7. (6 pts) True or false? If it is true, just say so. However, **if it is false, give a specific counterexample**.

a) T F x = c is equivalent to $x^3 = c^3$.

- b) T F If $\varepsilon > 0$ there exists k such that x > k implies $1/x < \varepsilon$.
- c) T F If $x_1^2 = x_2^2$, then $x_1 = x_2$.

8. (4 pts) Suppose this is all we know about *f*, which is defined on all real numbers. The maximum value over all real numbers *x* of f(x) is e^{-1} . What is the maximum value over all real numbers *x* of f(nx)? [For fixed $n \ge 2$.]

9. (10 pts) Theorem: "If x+y is irrational, then x is irrational or y is irrational." Prove it from the definition of "rational." [Do not assume any results we had in class. Use the definition.]

10. (3 pts) Give the contrapositive of the definition of "S is a subset of T."

11. (3 pts) Give the sentence-form definition of (set) union.

12. (12 pts) Read and use this definition to answer the true-false questions. Definition: *p* is a limes point of *S* iff for each $\delta > 0$ there are both a point of *S* and a point of *S*^c in the interval $(p - \delta, p + \delta)$.

- a) T F 2 is a limes point of (1, 5).
- b) T F 2 is a limes point of (2, 7].
- c) T F 2 is a limes point of {1, 2, 3, 4, 5}.

d) T F If c is a limes point of T, then c is a limes point of T^{c} .

e) Give a specific counterexample to "If S is a subset of T and p is a limes point of S, then p is a limes point of T.

13. (6 pts) Let f(x, y) = 3x + y if x < 4 and f(x, y) = x + 2y if $x \ge 4$.

Solve for x: f(x, x) = 14.