Exam on Chapters 1 and 2 of *Proof.*   Name:______________________

1. (10 pts) Suppose this is true: If $x \geq 8$, then $f(x) < 3$. Which of the following follow logically (FL)?

a) FL not FL If $x > 9$, then $f(x) < 4$.

b) FL not FL If $f(x) > 3$, then $x \leq 7$.

c) FL not FL If $f(x) > 4$, then $x < 9$.

d) FL not FL If $x = 10$, then $f(x) \leq 3$.

e) FL not FL $f(x) \neq 4$ when $x > 2$.

2. (10 pts) Suppose this is a fact: If $x < 4$ and $y > 1$, then $f(x, y) \geq 8$.
What can be deduced from that and this additional fact?

a) $y > 2$ and $f(x, y) < 7$.

b) $f(2, y) = 9$.

c) $f(x, 3) = 5$

d) $f(5, y) = 6$

e) $f(3, y) = 6$

3. (6 pts) Complete these logical equivalences the way they were completed as theorems in the text.

a) $(A \text{ or } B) \Rightarrow C$ is logically equivalent to

b) $A \Rightarrow (B \Rightarrow C)$ is logically equivalent to

c) Not(for all x, $H(x) \Rightarrow C(x)$) is logically equivalent to

4. (8 pts) True or false?

a) T F For each x in $(1, 7]$ there exists y in $(1, 7]$ such that $y > x$.

b) T F For each x in $(1, 7]$ there exists y in $(1, 7]$ such that $y < x$.

c) T F If $y > 5$ there exists $x > 10$ such that $x < 2y$.

d) T F If S is not bounded, then S^c is bounded.

5. (21 pts) Give the negation (in positive form) of

a) [Let f be given.] If $x < 5$, then $f(x) \leq 21$.

b) [Let f, and L be given.]
For $\varepsilon > 0$ there exists b such that $x > b$ implies $|f(x) - L| < \varepsilon$.

c) No quartic equation fails to have a solution.

d) [Let f be given.] For each m and z, $f(x) > m$ for some $x > z$.

e) [About piles of chips] At most two piles have less than ten chips.

f) If $b > 0$, then $ax^3 + bx = 10$ has at most one solution.

g) [About several teams] No team has more than one lineman under 250 pounds.

6. (6 pts) True or false? If it is true, just say so. However, if it is false, give a counterexample.

a) T F $|x + 1| > |x|$.

b) T F If a, b, and c are all greater than 0, then $ax^2 + bx + c > 0$.

c) T F For all a there exists b such that $x > b$ implies $x^2 > a$.

7. (4 pts) The nth odd number is $2n$-1. Here is a theorem: "$1 + 3 + 5 + ... + (2n-1) = n^2$."
Rewrite the theorem with problem-pattern "$1 + 3 + 5 + 7 + ... + k$" where k is odd.

8. (6 pts) Solve for y: $4x^2 + 3xy + 9x + 5y^2 + 2y = 91$.

9. (6 pts) Define ⊠this way: Let $x⊠y = 3y$ if $x \geq y$ and $x⊠y = 2y - x$ if $x < y$.
Solve for x in this equation: $5⊠x = 8⊠6$

10. (3 pts) We want a theorem that gives information about x and y and whether they are rational or irrational. Fill in the conclusion to this one "If $x+y$ is irrational, then ...."
[No proof required.]

11. (3 pts) Give the contrapositive of the definition of "b is an upper bound of S."

12. (3 pts) Give the sentence-form definition of (set) *intersection*.

13. (6 pts) Let S be given. Suppose you want to show "S is not bounded above." Expand that sentence to make all the quantifiers and quantified variables explicit so it will be clear exactly what you need to show. [That is, give the negation of "S is bounded above," with the quantifiers displayed.]

14. (8 pts) a) State the contrapositive of this: "If, for all $c > 0$, $x > d - c$, then $x \geq d$."
b) Then, prove it.