Solutions: Decimal numbers in these solutions were determined accurately and then rounded throughout to only 2 significant digits. That is enough for you to check your work. However, numbers here are not as precise as the ones you must display on your homework. Keep **at least 3 significant digits** to prove that you did the work.

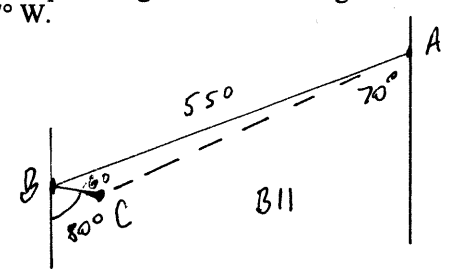
Chapter 8. Other Topics

Section 8.1. Vectors

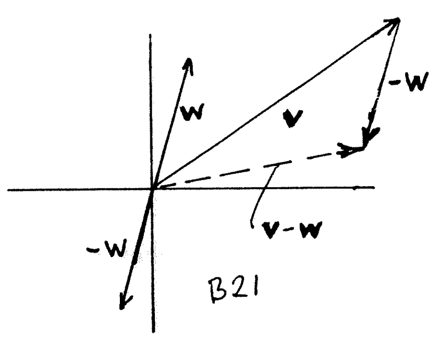
A1. $\langle 4, 9\rangle$ A3. $\langle 3.5, 8.0\rangle$ A5. $\langle 6, 8\rangle$ A7. $\langle 2, 4\rangle$
A9. $\langle -1, 3\rangle$ A11. $\langle 5, 7, 9\rangle$ A13. $\langle 105, 15\rangle$ A15. $500 \cos 20° = 470$.

^^^^^^^^

B1. Figure 3
B3. mag = 5.4, $\theta = 22°$
B5. mag = 10, $\theta = 180+37 = 217°$ (in III)
B6. mag = 100. $\theta = 30°$
B7. $\langle 8.7, 5\rangle$ B8. $\langle 71, 71\rangle$ B9. $\langle -17, 10\rangle$ B10. $\langle -.87, 4.9\rangle$
B11. Sketch a picture. The angle ABC with those two adjacent sides is 30 degrees. By SAS and the Law of Cosines, the ground speed is 500 mph. Using SSS the other angle is 3.45 degrees. So the actual bearing is 70 - 3.4 = S 67° W.



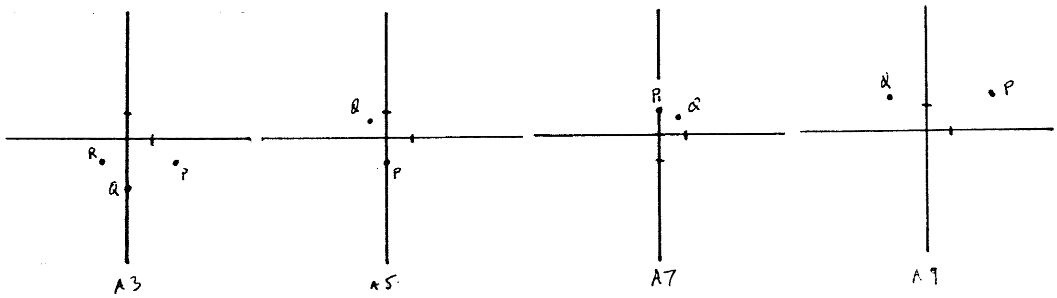
B13. See Figure 6B. $\langle c, 0\rangle + \langle -400 \sin 15°, -400 \cos 15°\rangle + \langle 0, d\rangle = \langle 0, 0\rangle$.
$c = 100$ pounds tension.
B15. $-v \sin 40° + w \sin 20° = 0$. $v \cos 40° + w \cos 20° - 100 = 0$.
$-.64v + .34w = 0$. $w = 1.9v$. $.77v + (1.9v)(.94) - 100 = 0$.
$v = 39$. $w = 74$.
B17. By symmetry, the force are the same. $2v \cos 20° = 100$. $v = 53$.
B19. a) 32 b) 0 c) $6+4 = \sqrt{(10)}\sqrt{(20)}\cos\theta$. $\cos\theta = .71$. $\theta = 45$ degrees.
d) $-2 + 2 = 0 = \sqrt{(5)}\sqrt{(5)} \cos\theta$. $\theta = 90°$
e) $\cos\theta = 0$ when $\theta = 90°$.
f) Make the dot product 0. $\langle -5, 1\rangle$ will do, and so will many others, proportional to this one.
g) $(a-c)^2 + (b-d)^2 = v^2 + w^2 - 2vw \cos\theta$, where v and w are the lengths of the vectors.
$(a-c)^2 + (b-d)^2 = a^2 + b^2 + c^2 + d^2 - 2vw \cos\theta$
$a^2 - 2ac + c^2 + b^2 - 2bd + d^2 = a^2 + b^2 + c^2 + d^2 - 2vw \cos\theta$
$ac - bd = vw \cos\theta$, as desired.
B21.



B23. a) Treated: (5+8+7+2+9+7)/6 = 6.33.
Untreated: (3+4+5+2)/4 = 3.5.
The treated average is higher.
b) v•w/10.

Section 8.2. Complex Numbers

A1. Add, separately, their real and imaginary components. [State 8.2.2]



A11. a) $9 + 8i$ b) $-1 - 6i$
A13. a) $13 + 13i$ b) $(2 + 3i)/(5 + i) = (2 + 3i)(5 - i)/[(5 + i)(5 - i)] = (13 + 13i)/26$
A15. a) $e^{i\pi/2}$ b) $e^{i\pi}$ c) $\sqrt{8}\, e^{i\pi/4}$
d) $\sqrt{17}\exp(i\tan^{-1}4)$ $[\tan^{-1} 4 = 1.3258]$ e) $2\, e^{2\pi i/3}$
A17. a) $\cos(\pi/2) + i\sin(\pi/2)$ b) $\cos\pi + i\sin\pi$ c) $\sqrt{8}[\cos(\pi/4) + i\sin(\pi/4)]$
d) $\sqrt{17}[\cos(\tan^{-1}4) + i\sin(\tan^{-1}4)]$ e) $2[\cos(2\pi/3) + i\sin(2\pi/3)]$
A19. a) $2.68 + 1.5i$ b) $-2.8 + 2.8i$
A21. $12\exp(i\pi) = -12$.
A23. $5\exp(5\pi i/12)$
A25. $(6 + 7i - 2)/13 = 4/13 + (7/13)i$
A27. π (= 180°). Rotating a number on the real line through π radians (180 degrees) changes its sign

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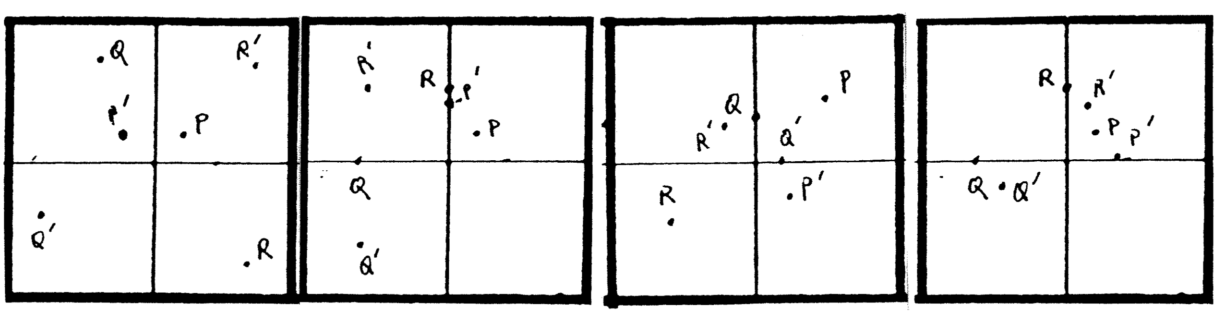
B1. Reproduce Figure 12.
B3. Use FOIL: State 8.2.3.

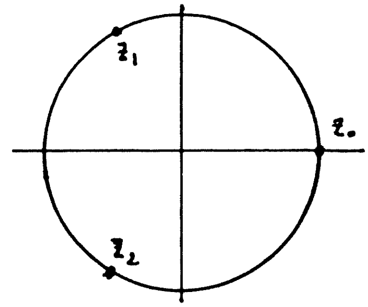
^^^^ The resulting points are labeled with primes: P', Q', and R'.



B5. Multiply by *i* **B7. Multiply by 1 + *i*.** **B9. Divide by 2*i***

B11. a) Expand away from the origin by a factor of c (if $c > 1$), and reversed direction if $c < -1$.
b) Let $w = re^{i\theta}$. Expand away from the origin by a factor of r (if $r > 1$) and rotate around the origin through an angle of θ.
B15. Use 8.2.16 (Multiply top and bottom by the complex conjugate of the bottom, and then multiply out the products)
B17. $i^2 = i$ times i. i is 1 unit from the origin at $\pi/2$. Multiplication by i produces a rotation by $\pi/2$. Rotating i by $\pi/2$ puts it at -1.
B19. $i = e^{i\pi/2}$. $iz = i\, r\exp(i\theta) = e^{i\pi/2}\, r\exp(i\theta) = r\exp(i(\theta+\pi/2))$. [8.2.12]
B21. $(-b + \sqrt{\ })(-b - \sqrt{\ })/(2a)^2 = (b^2 - (b^2 - 4ac))/(4a^2) = c/a$. Nice!
B25. To show $a/b = c$, show instead $a = bc$. This follows, in this case, from 8.2.12.

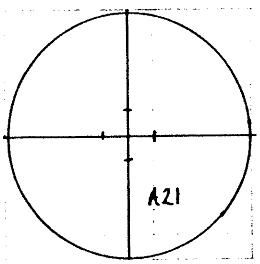
B27. a) and b): If $z = r e^{i\theta}$, then $z^2 = r^2 e^{2i\theta}$, and $z^n = r^n e^{in\theta}$.
c) Use $k = 0$, 1, and 2. $r^3 = 1$, so $r = 1$.
With $k = 0$, the solution is the usual $z = 1$.
With $k = 1$, $e^{2\pi i} = 1 = e^{i3\theta}$, so $2\pi = 3\theta$, and $\theta = 2\pi/3$.
With $k = 2$, $\theta = 4\pi/3$.

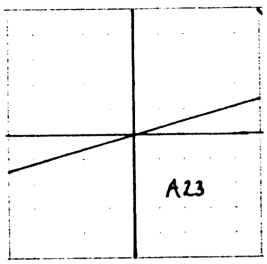


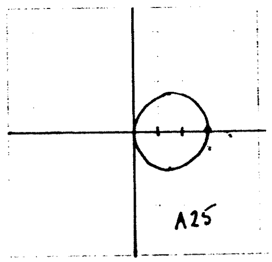
B29. Solve $z = z^2 - 1$. Use the Quadratic Formula.
$z^2 - z - 1 = 0$. $(1 \pm \sqrt{(1 + 4)})/2 = (1 \pm \sqrt{5})/2$.

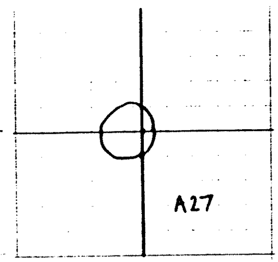
Section 8.3. Polar Coordinates

A1. $(\sqrt{8}, \pi/4)$ A3. $(4, \pi/2)$ A5. $(3, 0)$ A7. $(2, \pi/3)$
A9. $(0, 3)$ A11. $(2, 0)$ A13. $(2\sqrt{2}, 2\sqrt{2})$ A15. $(\sqrt{3}, 1)$
A17. $(2, -\pi)$, $(-2, 0)$, $(2, 3\pi)$ A19. $(-5, \pi)$, $(5, 2\pi)$, $(5, -2\pi)$

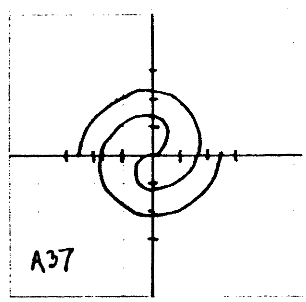








A29. 4
A31. The latter is 5 times as large. (Expanded away from the origin.)
A33. a) Bottom leaf (around the negative y-axis).
b) The leaf begins or ends when $\sin(3\theta) = 0$, so $3\theta = \pi$ or 2π or 3π ...
and $\theta = \pi/3$ begins the leaf and $\theta = 2\pi/3$ is the end.
c) I. d) r is negative for θ between $\pi/3$ and $\pi/2$ in I.
A35. $f(x) = 2 - x$
A37. Point symmetry through the origin



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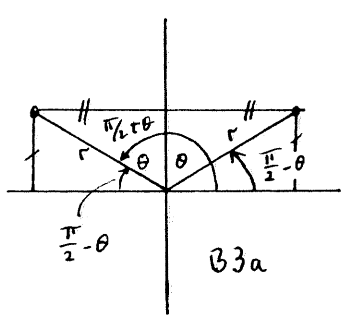
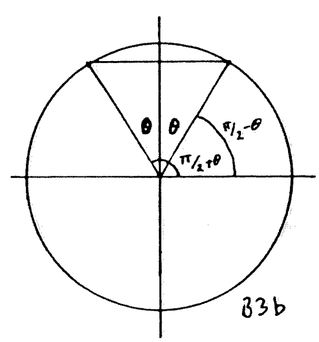
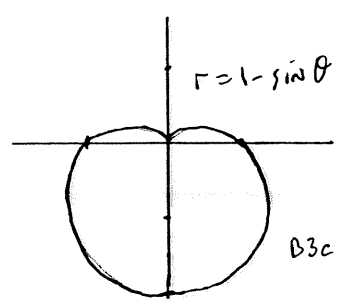
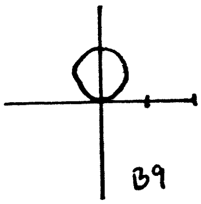
B1. Reproduce Fig. 8.
B3. a) & b) Figures below.
c) $(r, \pi/2 + \theta)$. $f(\sin(\pi/2 + \theta)) = f(\sin(\pi/2 - \theta))$,
so r is the same for θ and $-\theta$, which gives symmetry about the y-axis.

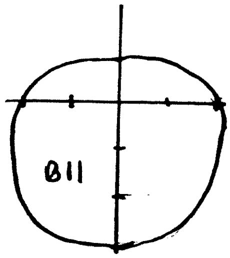






B7. $x = c = r\cos\theta$, $r = c/\cos\theta = c\sec\theta$.





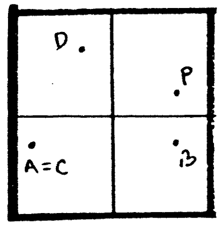
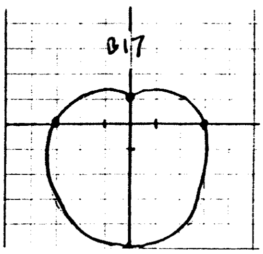
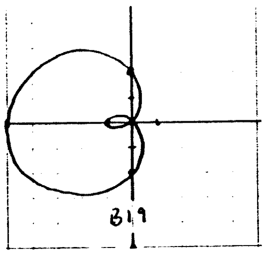


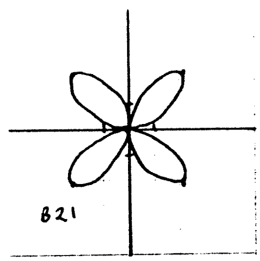
Figure for B5:

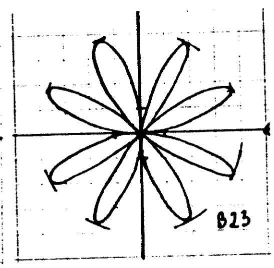
B13. Fig. 10, $\pi/2$.

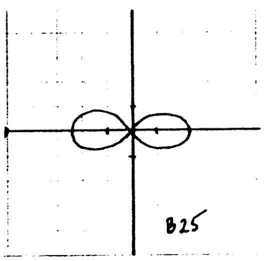
B15. Fig. 12, $\pi/2$.

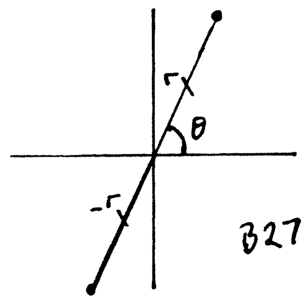












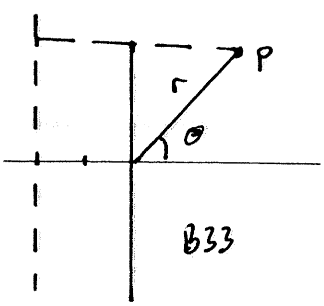
B27. b) $(-r, \theta)$. r^2 and $(-r)^2$ are equal.

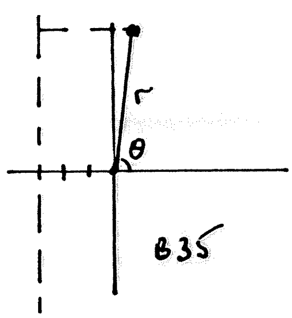
B29. Cos θ is symmetric about $\theta = 0$. Cos(2θ) is also. $\theta = 0$ is the x-axis. Therefore the graph of $f(\cos(2\theta))$ is symmetric about the x-axis. [Another, long winded, way: $\cos x = \cos(-x)$. $r = \cos(2\theta) = \cos(2(-\theta)) = r$. If (r, θ) is on the graph, so is $(r, -\theta)$.]

B31. Cos θ is symmetric about $\theta = 0, \pi, 2\pi$, etc. Therefore $\cos(3\theta)$ is symmetric about $\theta = 0, \pi/3, 2\pi/3$, etc. Therefore $f(\cos(3\theta))$ is symmetric about the lines $\theta = 0, \pi/3, 2\pi/3$, etc. The line in the first quadrant is $\theta = \pi/3$.

B33. See the picture below. a) $r = 2 + r\cos\theta$. b) $r - r\cos\theta = 2$. $r = 2/(1 - \cos\theta)$. c) a parabola

B35. See the picture below. $3 + r\cos\theta = r/2$. $6 + 2r\cos\theta = r$. $r = 6/(1 - 2\cos\theta)$.





B37. a) It is the same shape, rotated.
The graph of $r = f(\cos\theta)$ is the graph of $r = f(\sin\theta)$ rotated $\pi/2$ clockwise.
b) $\cos\theta = \sin(\theta + \pi/2)$. c) So cosine is sine shifted *left* (= lower) $\pi/2$, so $f(\cos\theta)$ will be the graph of $f(\sin\theta)$ with the argument smaller by $\pi/2$ (which is $\pi/2$ clockwise).

B39. It would not change shape, it would just become twice as large. For each fixed θ, r doubles, making similar triangles with the axes.

B41. Sin θ is symmetric about $\theta = \pi/2, 3\pi/2, \ldots, (2n+1)\pi/2$, for all n.
Therefore, $\sin(3\theta)$ is symmetric about $\theta = \pi/6, 3\pi/6 = \pi/2, \ldots, (2n+1)\pi/6$, for all n.
Therefore, $f(\sin(3\theta))$ is symmetric about the lines θ = those angles.

B43. Cos θ is symmetric about $\theta = 0, \pi, 2\pi, \ldots, k\pi$, etc.
Therefore $\cos(n\theta)$ is symmetric about $\theta = 0, \pi/n, 2\pi/n, \ldots k\pi/n$ etc.
Therefore $f(\cos(n\theta))$ is symmetric about the lines $\theta = 0, \pi/n, 2\pi/n, \ldots, k\pi/n$, for all k.

Section 8.4. Parametric Equations

A1. Fig. 4 has the circle traced at uniform speed. Fig. 5 also has the circle traced at uniform speed, but faster. In Fig. 6, the tracing accelerates.

A3. $y(t) = 0$ at the ground. Solve for $t > 0$.
$t = 6.3$ seconds. Then $x = 800(6.3) = 5000$ feet.
$x = 800t$ yields $t = x/800$. Substituting:
$y = -16(x/800)^2 + 100(x/800) + 5$
[quit here, but this may be simplified to $y = -.000025x^2 + .125x + 5$.]

A5. circle A7. circle A9. line A11. ray (half line)
A13. parabola A15. parabola A17. ellipse A19. line segment (not a whole line)
A21. circle
A23. The first goes clockwise from (0, 5), the second counterclockwise from (5, 0).
A25. One goes from (1, 5) to (3, 2) and beyond. The other goes in reverse.

*** Answers for the next group may vary. Parametric equations are NOT UNIQUE. ***

A27. $x = -1 + 3t, y = 6 - 2t$. A29. $x = 6\cos\theta, y = 6\sin\theta$.
A31. $x = 2\cos\theta, y = 8\sin\theta$.
A33. -2/3 A35. -1/3
A37. $t = (x - 4)/3$. $y = 2 - t$. $y = 2 - (x - 4)/3 = 10/3 - x/3$.
A39. $x^2 + y^2 = 49$.
^^^^^^^^

B1. They can represent non-functional relationships. They can express motion and when a point was reached.

B3. For any f such that its range is all real numbers,
$x = x_1 + af(t), y = y_1 + bf(t)$ is the line through (x_1, y_1).

B5. If the range is a subset of $[0, 2\pi)$, the graph will be part of a circle. For example,, if $f(t) = t$ on $[0, \pi]$, only the top half of the circle results.

B7. $x = a\cos t, y = b\sin t$.
B9. $x = 5\cos(\pi + t), y = 5\sin(\pi + t)$.
B11. $x = ct, y = 5ct$, at $t = 10$, $ct = 1$, so $c = 1/10$. $x = t/10, y = t/2$.
B13. $y/x = 7/3$. $y = (7/3)x$. A line segment from (-3, -7) to (3, 7).
B15. $t = x/50$. $y = -9.8(x/50)^2 + 40(x/50)$, a parabola.
B17. $y/x = 2, y = 2x$. This ray with domain $(0, \infty)$.
B19. Solve for $t > 0$ in $-16t^2 + 200t + 50 = 0$ and substitute it into $x(t) = 2500t - 300t^{1.5}$.
It hits at 18,000 feet.
B21. $x = \cos t, y = t$.
B23. $x = t^4 - 5t^2, y = t$.
B25. $x = t^3 - 3t, y = t^3 - 5t$.
B27. $x = 8\cos\theta + \cos(12\theta), y = 8\sin\theta + \sin(12\theta)$.
[This curve is NOT realistic for the Earth's moon, since the ratio 8 to 1 is so far wrong. There are no cusps on the real path of the moon, which is much closer to circular around the sun.]

Chapter 9. Conic Sections

Section 9.1. Conic Sections: Parabolas

A1. $y = x^2/[4(1/4)]$; $V = (0, 0)$; axis: $x = 0$; F: $(0, 1/4)$; dir: $y = -1/4$.
Has the usual graph of x^2.

A3. $y = (x - 1)^2/[4(1/4)] + 2$; $V = (1, 2)$; axis: $x = 1$; F: $(1, 9/4)$; dir: $y = 7/4$.
The usual graph of x^2, shifted right 1 and up 2.

A5. $y = x^2 + 4x = x^2 + 4x + 4 - 4 = (x + 2)^2/[4(1/4)] - 4$;
$V = (-2, -4)$; axis: $x = -2$; F: $(-2, -15/4)$; dir: $y = -17/4$.
The usual graph of x^2, shifted left 2 and down 4.

A7. $y = 2 - x^2 = x^2/[4(-1/4)] + 2$; $V = (0, 2)$; axis: $x = 0$; F: $(0, 7/4)$; dir: $y = 9/4$.
The graph of x^2 fliped upside down and then up 2.

A9. $y = 2x^2 - 8x - 5 = 2(x^2 - 4x) - 5 = 2(x^2 - 4x + 4) - 13 = (x - 2)^2/[4(1/8)] - 13$.
$V = (2, -13)$; axis: $x = 2$; F: $(2, -13 + 1/8)$; dir: $y = -13 - 1/8$.
The graph of x^2 made twice as tall and shifted right 2 and down 13.

A11. $x = y^2 = y^2/[4(1/4)]$; $V = (0, 0)$; axis: $y = 0$; F: $(1/4, 0)$; dir: $x = -1/4$.
The graph of $y = \pm\sqrt{x}$.

A13. $x = (y - 2)^2 + 3 = (y - 2)^2/[4(1/4)] + 3$;
$V = (3, 2)$; axis: $y = 2$; F: $(13/4, 2)$; dir: $x = 11/4$.
The graph of $y = \pm\sqrt{x}$ shifted right 3 and up 2.

A15. $x^2 + 2x - y = 5$. $y = x^2 + 2x - 5 = (x^2 + 2x + 1) - 1 - 5 = (x + 1)^2/[4(1/4)] - 6$
$V = (-1, -6)$; axis: $x = -1$; F: $(-1, -23/4)$; dir: $y = -25/4$.
The graph of x^2 upside down and shifted left 1 and down 6.

A17. $y = x^2/4$ A19. $x = y^2/4$

A21. $y = (x - 3)^2/[4(6)] + 4$. A23. $y = x^2/[4(2)]$

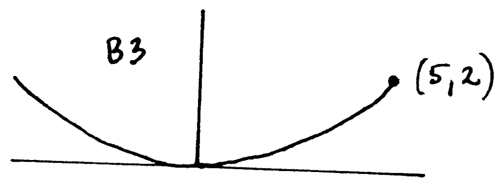
A25. $y = (x - 1)^2/4 + 1$. A27. $y = (x - 2)^2/[4(5)] - 5$.

A29. $x = cy^2$, for some c. $4 = c1^2$, so $c = 4$. $x = 4y^2$.

^^^^^^^^

B1. 9.1.1, Figure 3.

B3. See the picture. With the axis through the middle and the vertex on the dish, the equation is $y = cx^2$, for some c. Fit the known point.
$2 = c5^2$. $c = 2/25$.
$y = (2/25)x^2 = x^2/[4a]$. $4a = 25/2$ so $a = 25/8$.
The focus is 25/8 feet from the bottom of the dish.

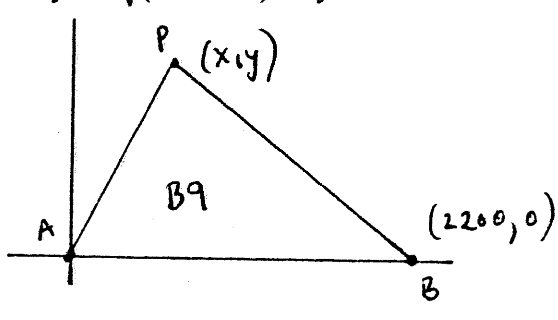


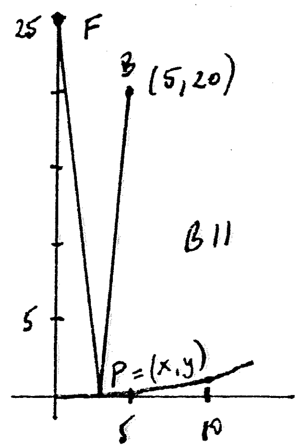
B5. Draw a picture similar to the one for B3. With the axis through the middle and the vertex on the dish, the equation is $y = cx^2$, for some c. Fit the known point.
$6 = c6^2$, so $c = 1/6$. $y = (1/6)x^2 = x^2/[4a]$, so $a = 3/2$.
The focus is 3/2 inch from the bottom of the dish.

B7. $y = x^2/[4(18)]$. $y = 3$ at $3 = x^2/72$. $x = 15$.
The dish is 29 inches wide.

B9. The location of your axis system may vary. If you use one of the points as the center (as in the picture), you would get
$AP + PB = 3300$, because sound travels 3300 feet in 3 seconds.

$$\sqrt{x^2 + y^2} + \sqrt{(2200 - x)^2 + y^2} = 3300.$$





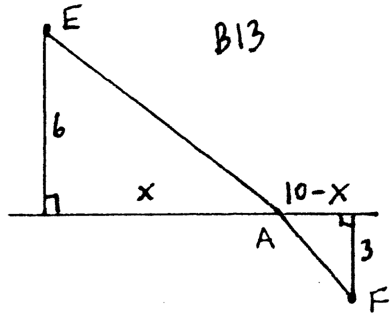
B11. See the picture above. Distance =

$$\sqrt{(5-x)^2+(20-y)^2}+\sqrt{x^2+(y-25)^2}$$

$$=\sqrt{(5-x)^2+\left(20-\frac{x^2}{100}\right)^2}+\sqrt{x^2+\left(\frac{x^2}{100}-25\right)^2}$$

Graph it. Its minimum occurs when $x = 5.00$

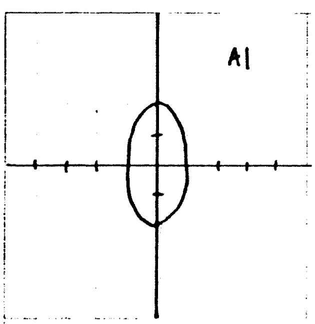
B13. $EA = \sqrt{(36 + x^2)}$. $AF = \sqrt{(9 + (10-x)^2)}$.
Time is proportional to
$\sqrt{(36 + x^2)} + 1.3\sqrt{(9 + (10-x)^2)}$.
The domain is [0, 10]. Graph it.
The minimum occurs at $x = 7.8$.
b) The eye sees the light coming from A.
Extending line EA would yield a line that goes above the fish.

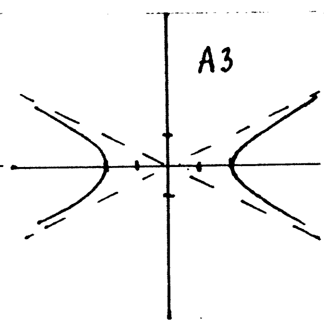


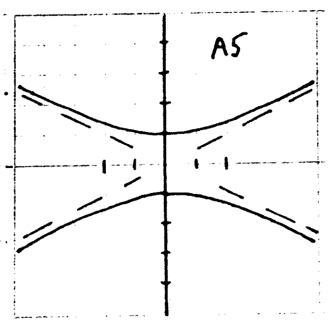
B15. Let $x^2 + y^2 = 64$. Light from the origin reflects back through the origin from all points. Since the figure is a circle, all points are equidistant.

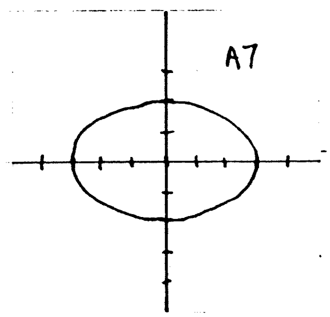
Section 9.2. Ellipses and Hyperbolas

A1. ellipse, $a = 1$, $b = 2$, $c = \sqrt{3} = 1.7$, center (0, 0).
A3. hyperbola, $a = 2$, $b = 1$, $c = \sqrt{5} = 2.2$, center (0, 0).

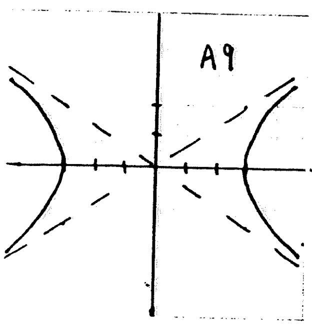


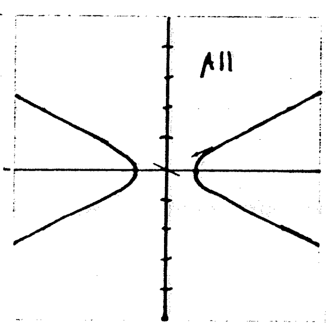


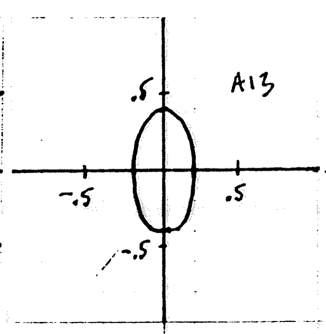


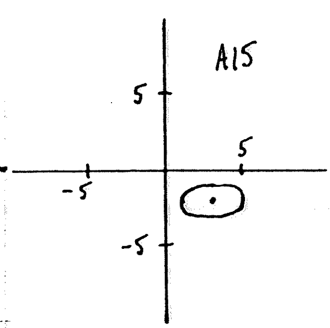


A5. hyperbola, $a = 2$, $b = 1$, (vertical axis), $c = \sqrt{5} = 2.2$, center (0, 0).
A7. $x^2/3^2 + y^2/2^2 = 1$, ellipse, $a = 3$, $b = 2$, $c = \sqrt{5} = 2.2$, center (0, 0).
A9. $x^2/3^2 - y^2/2^2 = 1$, hyperbola, $a = 3$, $b = 2$, $c = \sqrt{13} = 3.6$, center (0, 0).
A11. $x^2/1^2 - y^2/(1/2)^2 = 1$, hyperbola, $a = 1$, $b = 1/2$, $c = 1.1$, center (0, 0).





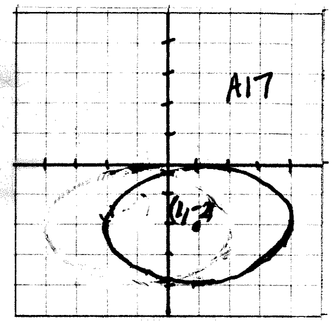




A13. $x^2/(1/5)^2 + y^2/(2/5)^2 = 1$, ellipse, $a = 1/5$, $b = 2/5$, $c = .35$, center $(0, 0)$.

A15. $(x - 3)^2/2^2 + (y + 2)^2/1^2 = 1$, ellipse, $a = 2$, $b = 1$, $c = \sqrt{5} = 2.2$, center $(3, -2)$.

A17. $4(x^2 - 2x + 1 - 1) + 9(y^2 + 4y + 4 - 4) = -4$.
$4(x - 1)^2 + 9(y + 2)^2 = -4 + 4 + 36 = 36$.
$(x - 1)^2/3^2 + (y + 2)^2/2^2 = 1$,
ellipse, $a = 3$, $b = 2$, $c = \sqrt{13} = 3.6$, center $(1, -2)$.



A19. $c = 2$, $a = 3$. $c^2 + b^2 = a^2$, so $b = \sqrt{5}$. $x^2/3^2 + y^2/(\sqrt{5})^2 = 1$.

A21. $b = 4$, $a = 2$. $(x - 1)^2/2^2 + (y - 3)^2/4^2 = 1$.

A23. $a = 1$, $c = 2$. $a^2 + b^2 = c^2$, so $b = \sqrt{3}$. $x^2/1^2 - y^2/(\sqrt{3})^2 = 1$.

A25. $b = 4$, $c = 5$. $a^2 + b^2 = c^2$, so $a = 3$.
$(y - 5)^2/4^2 - (x - 1)^2/3^2 = 1$.

A27. hyperbola A29. circle A31. parabola A33. circle

A35. hyperbola

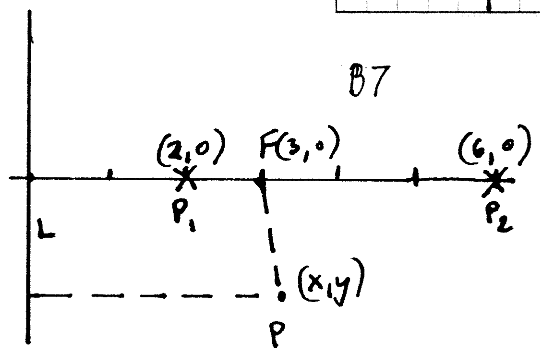
^^^^^^^^

B1. Reproduce Figure 9.

B3. Reproduce Figure 17.

B5. $\pm d/c$

B7. a) (2, 0), (6, 0)
b) (3, 1.5), (3, -1.5)
$x/2 = \sqrt{[(x - 3)^2 + y^2]}$
$x^2 = 4(x^2 - 6x + 9 + y^2)$,
$0 = 3x^2 - 24x + 36 + 4y^2$,
$0 = 3(x^2 - 8x + 16 - 16) + 36 + 4y^2$
$0 = 3(x - 4)^2 + 4y^2 - 12$.
$(x - 4)^2/2^2 + y^2/(\sqrt{3})^2 = 1$. (Ellipse)



B9. a) $e = c/a = 1/2$. $c = a/2$. $b^2 + c^2 = a^2$, so $b^2 + (a/2)^2 = a^2$, and $b^2 = 3a^2/4$.
$b = (\sqrt{3}/2)a$. $a/b = 2/\sqrt{3} = 1.2$.
b) $a/b = 2$. $a/2 = b$. $c^2 = a^2 - b^2 = a^2 - (a/2)^2 = 3a^2/4$. $c = (\sqrt{3}/2)a$.
$c/a = \sqrt{3}/2 = .87$.

B11. Slope $= b/a$. $e = c/a$. $a^2 + b^2 = c^2$. $c = ae$. $a^2 + b^2 = e^2a^2$. $b^2 = a^2(e^2 - 1)$.
$b/a = \sqrt{(e^2 - 1)}$.

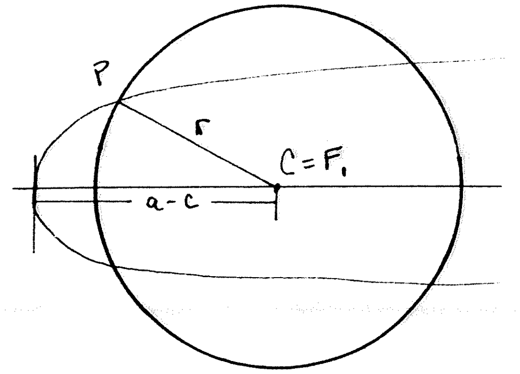
B13. Let $a > b$, so the ratio $r = a/b$. We want a formula for $e = c/a$.
$c^2 = a^2 - b^2$. $b = a/r$. $c^2 = a^2 - (a/r)^2 = a^2(1 - 1/r^2)$.
$e = c/a = \sqrt{(1 - 1/r^2)}$ [$= \sqrt{(r^2 - 1)}/r$. For large r, e is nearly r.]

B15. Figure 19 (c/a is greater in Figure 19).

B17. Although there are some minus signs in early steps, squaring eliminates them. The difference comes after you get to the equation with "$a^2 - c^2$" as factors of two terms. For hyperbolas, $c > a > 0$, so $c^2 > a^2$. Let $b^2 = c^2 - a^2$ (instead of $a^2 - c^2$). Now there are two minus signs, but the rest continues as before.

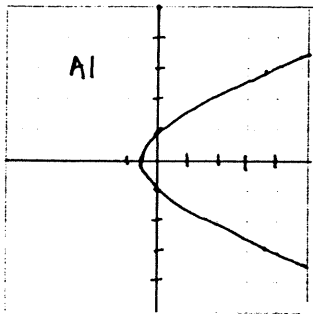
B19. We know the time for the reflected sound to travel is 2 seconds plus the time between the first sound and the second. Here, the total is 2½ seconds, and the distance the reflected sound travels is 2200 + 550 feet = 2750 feet.
So $2a = 2750$ and $2c = 2200$. We can create the equation from these facts.

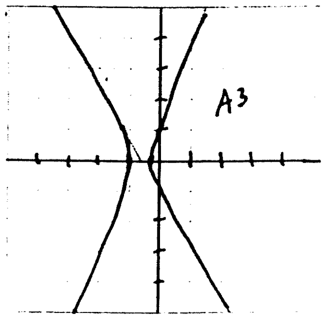
B21. No (unless the circle and the ellipse are the same). [Other arguments are possible.] Let the axis system be so that the major axis of the ellipse is the x-axis, so $a > b$. Consider the picture. If there are two intersections on the left side of the circle as well as the right, the picture holds and $a - c > r$, so $a > r + c$.
Note $PF_2 \le r + 2c$ (a line is the shortest distance between two points.)
Then $F_1P + PF_2 \le r + r + 2c = 2r + 2c < 2a$ (if there are intersections on the left). But that total is $2a$ for an ellipse, so there cannot be intersections on the left.

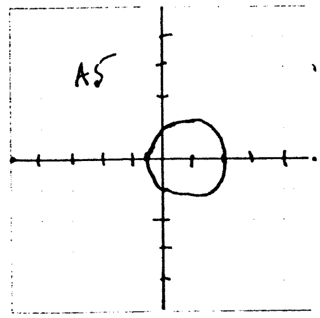


Section 9.3. Polar Equations of Conic Sections

A1. parabola, $e = 1$.
A3. hyperbola, $e = 2$.







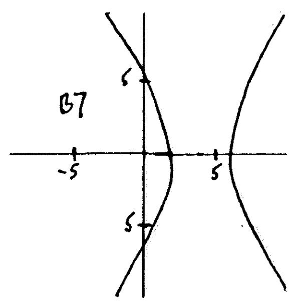
A5. ellipse, $e = .5$.
A7. parabola, $e = 1$, dir: $x = 5$.
A9. $r = (7/3)/(1 - (2/3)\sin\theta)$. ellipse, $e = 2/3$, dir: $y = -7/2$.
A11. $r = (11/9)/(1 + (10/9)\sin\theta)$. hyperbola, $e = 10/9$, dir: $y = 11/10 = 1.1$.
A13. $r = 2/(1 - \sin\theta)$ $[= -2/(1 + \sin\theta)]$.
A15. $r = 3(.95)/(1 + .95\cos\theta)$.
A17. $r = 6/(1 + 1.5\sin\theta)$.
^^^^A19-22 use 9.3.7.
A19. $ed = a(1 - e^2) = .39(1 - .21^2) = .37$.
$r = .37/(1 - .21\cos\theta)$.
A21. $ed = a(1 - e^2) = .72(1 - .0068^2) = .72$.
$r = .72/(1 - .0068\cos\theta)$.
A23. $ed = a(1 - e^2)$. $ed = 4(1 - .7^2)$. $r = ed/(1 - e\cos\theta)$ by 9.3.2. $r = 2.0/(1 - .7\cos\theta)$
^^^^^^^^

B1. III, I, IV, II
For θ near 0 in I, $1 - 2\cos\theta < 0$, so $r < 0$ which yields points in III.

B3. The minimum of $1 - .8\cos\theta$ is .2. The minimum of $1 - .95\cos\theta$ is .05. Dividing by .05 yields a number four times as large as dividing by .20. So, the numerator would need to be four times as large to yield the same maximum value.

B5. a) symmetric about the x-axis, because b) $\cos\theta$ is symmetric about $\theta = 0$, which is the equation of the x-axis.

B7. a) $r = 6/(1 + 2\cos\theta)$.
c) $r = -6/(1 - 2\cos\theta)$.
d) & e) the graph is the same as in (b).
f) $\cos(\pi + \theta) = -\cos\theta$. Suppose (r, θ) is on the graph of (a).
Then at angle $\pi + \theta$ the value of the denominator in (c) would be the same as in (a), and the numerator would be its negative.
So $(-r, \pi + \theta)$ is on the graph of (c). But that is the same point.
Only the notation has changed!
The same points occur with different θ values.



B9. "$\sin\theta = \cos(\theta - \pi/2)$." The graph using "$\sin\theta$" could be rewritten using "$\cos(\theta - \pi/2)$," which would assume the same r values when θ is $\pi/2$ higher. So it (with sine) is rotated counterclockwise by $\pi/2$.

B11. $1 - 3\cos\theta = 0$ when $\cos\theta = 1/3$ and $\theta = 1.2$ or 5.1. $[0, 1.2)$ union $(5.1, 2\pi]$.

B13. We want the slopes $\pm b/a$. $e = c/a$ [9.3.8]
$a^2 + b^2 = c^2$ [9.2.10] Dividing by a^2,
$1 + (b/a)^2 = (c/a)^2 = e^2$. $(b/a)^2 = e^2 - 1$. $m = \pm\sqrt{(e^2 - 1)}$.

B15. $b/a = 1/4$. We want $e = c/a$ [9.3.8]
$a^2 + b^2 = c^2$ [9.2.10] Dividing by a^2,
$1 + (b/a)^2 = (c/a)^2 = e^2$. $1 + (1/4)^2 = e^2$. $e = 1.0$.

B17. Let $a > b$, so the ellipse is horizontal. We want b/a, given $e = 1/2$.
For an ellipse with eccentricity e, $e = c/a$ [9.3.8]
$c^2 + b^2 = a^2$ [9.2.5]. Dividing by a^2
$(c/a)^2 + (b/a)^2 = 1$. $e^2 + (b/a)^2 = 1$. $(b/a)^2 = 1 - (1/2)^2$ $b/a = .87$.

B19. Let $a > b$, so the ellipse is horizontal. $b/a = 1/3$. We want $e = c/a$.
$c^2 + b^2 = a^2$ [9.2.5]. Dividing by a^2
$(c/a)^2 + (b/a)^2 = 1$. $e^2 + (1/3)^2 = 1$. $e = .94$.

B21. $e = .1$. $r = ed/(1 - .1 \cos \theta)$, so ed is about 5, so d is about 50.
The directrix is about 50 units from the origin (any direction is possible).

B23. The former has $ed = .02d$ is about 1, so d is about 50.
The latter has $ed = .01d$ is about 1, so d is about 100.
The directrix of the latter is twice as far from the origin.

B25. When $\theta = 0$, $r = -100$. $a = ed/|1 - e^2|$ [9.3.6] $= 1/|1 - 1.01^2|$, so $a = 50$.
The center is, therefore, along the x-axis at $x = -100 + 50 = -50.25$.
From B13, $m = \pm\sqrt{(e^2 - 1)} = \pm\sqrt{(1.01^2 - 1)} = \pm 0.14$.

B27. $e = 0.7$. $a = ed/|1 - e^2|$ [9.3.6] $= 2/(1 - 0.7^2)$, so $a = 3.9$.
$e = c/a$, so $c = ae = 0.7(3.9) = 2.7$. $b^2 = a^2 - c^2$ [9.2.5], so $b = 2.8$.
When $\theta = 0$, $r = 2/(1 - .7) = 6.7$. Center on the x-axis at x-value $6.67 - a = 2.7$.

B29. $e = 1/2 = c/a$. $a^2 - c^2 = b^2$. Dividing by a^2, $1 - (c/a)^2 = (b/a)^2$.
$3/4 = (b/a)^2$. $b/a = .87$. Given $b = 3$, $a = 3.5$, and $c = 1.7$.
Center (1.732, 0). So the equation is $(x - 1.7)^2/3.5^2 + y^2/3^2 = 1$.

B31. $x = (5 \cos \theta)/(2 - 4 \cos \theta)$, $y = (5 \sin \theta)/(2 - 4 \cos \theta)$.

B33. Let the directrix be $y = -d$. $\sqrt{(x^2 + y^2)} = d + y$. $x^2 + y^2 = d^2 + 2dy + y^2$.
$x^2 = d^2 + 2dy$. $y = (x^2 - d^2)/2d$.

B35. a) Let the directrix be $x = -d$. $\sqrt{(x^2 + y^2)} = e(x + d)$.
b) $x^2 + y^2 = e^2(x + d)^2$. $y = \pm\sqrt{[e^2(x + d)^2 - x^2]}$
c) $y = \pm\sqrt{[4(x + 3)^2 - x^2]}$