Precalculus. Final, Spring 2013 Name_____
Instructor, Section number, or class time of day_____
SHOW SUPPORTING WORK!! Little or NO CREDIT will be given unless appropriate supporting work is displayed (except on one-step problems). **You must use algebraic methods** whenever possible. If (and only if) no algebraic method works, guess-and-check is legal and expected.

Formulas

The compound interest formula is $A = P(1 + r/k)^{kt}$.
The continuous-compounding model is: $A = Pe^{rt}$.
The doubling-time model is: $P(t) = P(0)\,2^{t/d}$.
The half-life model is: $A(t) = A(0)(1/2)^{t/h}$.
The exponential-growth model is: $P(t) = P(0)\,e^{kt}$.

#	Points	Score
1	5	
2	6	
3	5	
4	5	
5	15	
6	4	
7	10	
8	8	
9	10	
10	8	
11	9	
12	5	
13	10	
algebra	100	
14	24	
15	18	
16	10	
17	8	
18	8	
19	6	
20	6	
21	12	
22	8	
trig	100	
total	200	

1. Let $d = -1.72$. Find, with at least three correct significant digits $\dfrac{-3+\sqrt{d^2+6.2}}{(\pi-2)(\pi-2.7)}$ .05214

2. Let $f(x) = 1/(x+1)$. Find and simplify $f(f(x))$

$= \dfrac{1}{\left(\frac{1}{x+1}\right)+1} = \dfrac{1}{\frac{1+x+1}{x+1}} = \dfrac{x+1}{2+x}$

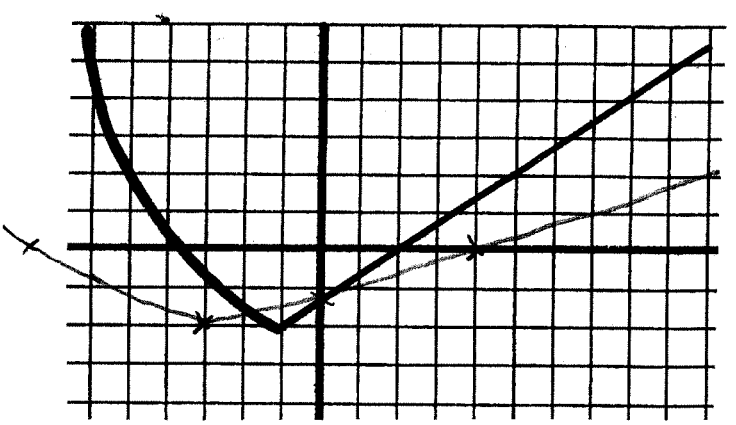
3. The cost of photocopies is 5 cents each for the first ten copies and 3 cents each for copies after the first ten. Give the formula for the cost, $C(n)$, of n copies when $n > 10$.

$C(n) =$ $50+3(n-10)$ $[= 20+3n]$

4. State the **algebraic formulation** of the method used to evaluate these expressions in terms of simpler operations. Do not find the numbers, state the one method, in symbols. Do not use English.

5/(7/3), $\dfrac{x^2}{\left(\frac{x+1}{2x}\right)}$, $\dfrac{25}{\left(\frac{5}{50}\right)}$ $\dfrac{a}{\left(\frac{b}{c}\right)} = \dfrac{ac}{b}$

5. Here is a representative graph of $f(x)$. Grid lines are one unit apart.



a) Solve for x: $f(x) = x$

$x = -6.7$

b) Solve for x: $f(x) = 2f(5)$

$f(x) = 2(2) = 4$

$x = -6.9$ or $x = 7.9$

c) Sketch, on the same axis system, $y = f(x/2)$.
[Get points where the axes are crossed right.]

6. You get a graph in the window [0, 10] by [0, 10] and find it looks too tall and too close to the y-axis. You want it to look shorter and wider. You have the best chance of getting what you want if you change the window to [pick one]

a) [0, 5] by [0, 5] (b) [0, 5] by [0, 20] c) [0, 20] by [0, 5] d) [0, 20] by [0, 20]

7. Consider the graph of $y = x^2$ and the points on it where $x = a$ and $x = b$.

a) Find, in point-slope form, the line through those two points.

b) Simplify the slope as much as appropriate (but maintain point-slope form). [Do not use $y = mx + b$ form.]

(a, a^2) (b, b^2)

$$y - a^2 = \frac{b^2 - a^2}{b - a}(x - a)$$

$$y - a^2 = (b + a)(x - a)$$

8. Solve for y [show appropriate work, of course]: $3x^2 + 7xy + 5x + 2y^2 - y = 400$

Use the Quadratic Formula

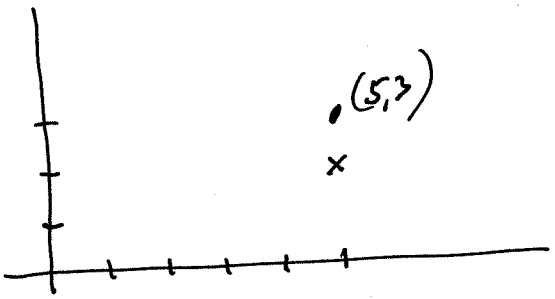
with "a" = 2 "b" = $7x - 1$ "c" = $3x^2 + 5x - 400$

9. A point in the plane is twice as far from the x-axis as it is from (5, 3).

a) Sketch a picture of that information.

b) Use guess-and-check to find one such point. (x, y)

c) SET UP and equation that could be used to locate all such points. [Do not solve it or even bother to simplify if you have your equation right.]



$(5, 2)$, is one. There are others.

distance of (x, y) from the x-axis: y in 1st quadrant

" " " " $(5, 3)$, is $\sqrt{(x-5)^2 + (y-3)^2}$

$$y = 2\sqrt{(x-5)^2 + (y-3)^2}$$

10. A stock went up 60 percent, total, in the two years 2011 and 2012. In 2012 alone it went 40 percent.
a) How much did it go up per year, on average, over those two years?
b) How much did it go up in 2011?

$(1.6)^{1/2} = 1.2649$ up 26.49%/year

$(1+r)(1.4) = 1.6$

$r = .1428$ up 14.28%

11. Short answer. [No work required.]
a) A substance decays with half-life 25 seconds. How long will it be until there is one-eighth the original amount?

3 half-lives = 75 seconds

b) Rewrite "$|x - 5| < 2$" in the form "$a < x < b$".

$-2 < x-5 < 2$
$3 < x < 7$

c) Rewrite this in "exponential form": $\log_2 c = k$.

$2^k = c$

12. Find k such that $e^{kt} = 2^{t/d}$ for all t.

take logs $\quad kt = \frac{t}{d} \ln 2 \quad k = \frac{\ln 2}{d}$

13. Using the doubling-time model, when will there be 100,000 bacteria if there are now 2000 bacteria and the doubling-time is 35 minutes?

$100{,}000 = 2000\left(2^{t/35}\right)$ solve for t

Part II: Trigonometry. **Set your calculator to DEGREE mode** to start. Switch to radian mode when appropriate. For your information: Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.
Law of Sines: $(\sin A)/a = (\sin B)/b$. $\sec\theta = 1/(\cos\theta)$, $\csc\theta = 1/(\sin\theta)$, $\cot\theta = 1/(\tan\theta)$.

14. Short answer:

a) Give θ in degrees, $90° < \theta < 180°$ (in the **second** quadrant), such that $\tan\theta = -2$.

$\tan^{-1}(-2) = -63.42°$ add $180°$ to get $116.56°$

b) Solve $\sec\theta = 1.5$ for θ in degrees in the first quadrant.

$\frac{1}{\cos\theta} = 1.5$ $48.19°$

c) If $\theta = 8\pi/5$, give the reference angle of θ.

$2\pi - \frac{8\pi}{5} = \frac{2\pi}{5}$

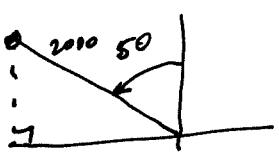
d) If you want to solve a triangle and you have the information from Angle-Side-Angle, what part do you find next?

third angle

e) How many degrees are in one radian, **exactly**. [Do not give a decimal answer.]

$\frac{1}{2\pi} = \frac{\theta}{360}$ $\theta = \frac{360}{2\pi}$

f) From point A, point B is 2000 feet, N 50° W. How far west of A is B?



$2000 \sin 50° = 1532'$

15. The unit-circle figure has two equal angles labeled θ and two right angles, as marked.

a) If θ = 32 degrees, find AD = $\sin 64 = .89879$

b) If θ = 32 degrees, find BC =

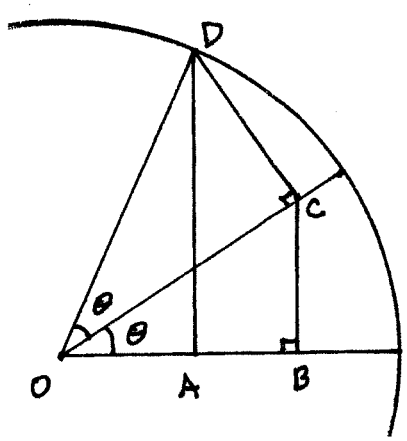
$OC = \cos\theta$

$BC = \sin\theta\cos\theta$ $BC = .4494$

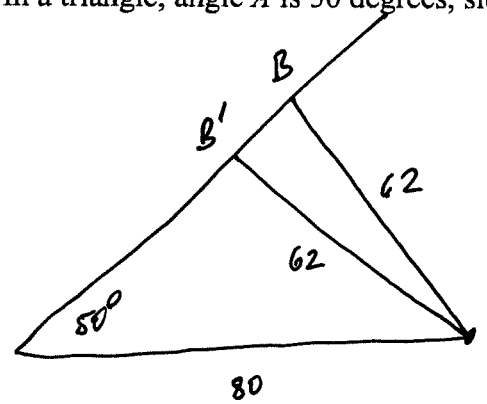
c) If θ is unknown and OB = 0.69, find θ.

$OB = \cos\theta\cos\theta = \cos^2\theta = .69$

$\theta = 33.83°$



16. In a triangle, angle A is 50 degrees, side b is 80, and side $a = 62$. Find angle B.

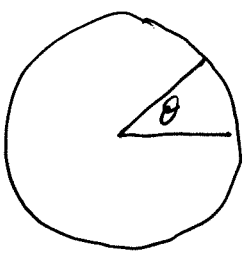


$\frac{\sin B}{80} = \frac{\sin A}{62}$

$\sin B = .98844$

$B = 81.28^\circ$ or $B = 98.719^\circ$

17. **Derive** the formula for the area of a sector if the central angle is in degrees. [A sector is a pie-shaped piece of a circle. The formula alone is worth little. Demonstrate to us you know why it is the right formula.]



$\frac{A}{\pi r^2} = \frac{\theta^\circ}{360^\circ}$ $\quad A = \frac{\theta^\circ \pi r^2}{360}$

proportions!

18. Sketch and fully **label** an excellent and illuminating unit-circle picture to determine and illustrate the usual trig identity for $\sin(\theta + \pi/2)$.

a) **Label**, in the proper locations, at least these:
θ, $\theta + \pi/2$ (these two angles must have labeled arcs),
and $\sin(\theta + \pi/2)$ (the location of this length must be labeled).

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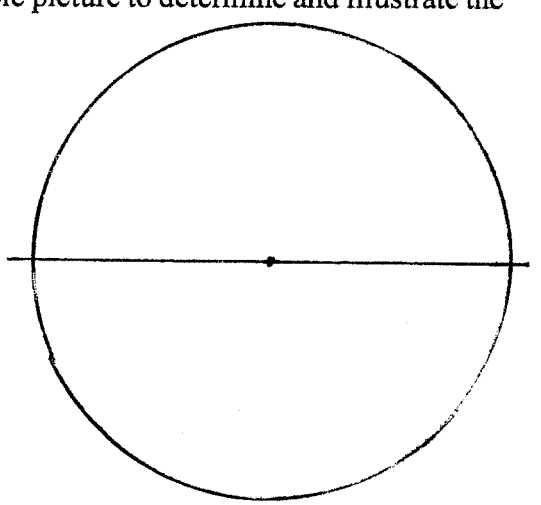
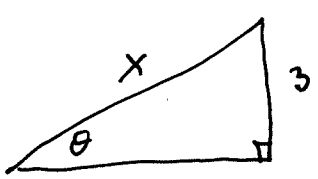
b) Give the usual trig identity for $\sin(\theta + \pi/2)$.
[The identity alone will be worth little.
The picture will be marked on how illuminating and how completely **labeled** it is.]



19. When the angle is in the first quadrant and $\sin(\theta) = 3/x$, express $\tan(\theta)$ in terms of x.

Use:



p 414-5

Find the θ here

20. Here are identities you might wish to use in this problem:

(7.3.3A) $\sin 2\theta = 2(\sin\theta)(\cos\theta)$

(7.3.3B) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(7.3.3C) $\cos 2\theta = 1 - 2\sin^2\theta$

(7.3.3D) $\cos 2\theta = 2\cos^2\theta - 1.$

Derive the identity for $\sin^2\theta$ that we derived from one of these.

[Do not derive the one from $\sin^2\theta + \cos^2\theta = 1$. Derive the other one we derived from one of these. Giving the identity without the steps is worth zero.]

From 7.3.3C $\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta - 1 = -2\sin^2\theta$

$\frac{\cos 2\theta - 1}{-2} = \sin^2\theta \quad = \quad \frac{1-\cos 2\theta}{2}$

21. Solve algebraically for θ in degrees in the first quadrant. [Show work! You must show clear supporting algebraic work.]

a) $\tan\theta = 3\sin\theta.$

$\frac{\sin\theta}{\cos\theta} = 3\sin\theta \qquad \frac{1}{\cos\theta} = 3 \qquad \theta = 70.53°$

b) $\cos(2\theta) = \sin\theta - 0.3$

$1 - 2\sin^2\theta = \sin\theta - 0.3$

Use Quadratic Formula $\qquad \theta = 36.45°$

22. See the figure. If the arc from B to P is six times as long as the distance from A to O, find the central angle θ.

$\overset{\frown}{BP} = 6\,\overline{AO}$

$\theta = 6(-\cos\theta)$

solve with calculator in radians

$\theta = 1.89$ radians