

Precalculus. Final, Fall 2013 Name _____

Instructor, Section number, or class time of day _____

SHOW SUPPORTING WORK!! Little or NO CREDIT will be given unless appropriate supporting work is displayed (except on one-step problems).

You must use algebraic methods whenever possible. If (and only if) no algebraic method works, guess-and-check is legal and expected.

1. (Short answer) Let $k = -1.23$.

Evaluate
$$\frac{\pi - \sqrt{k^2 + 2(5.67 - 3.2)}}{1.3 + 1.975} =$$

[Give at least three correct significant digits.]

2. Simplify and then solve for p : For all x , $(x^3)^p = x^{10}\sqrt{x}$.

3. True or false (no reason required):

T F The "x" in " $3x + 5 = 17$ " is not a placeholder.T F The equation " $y = mx + b$ " has two parameters.4. Let $f(x) = x^2$. Find and simplify
$$\frac{f(x+h) - f(x)}{h}$$
5. The equation $x^2 + y^2 + 5y + 3 = 40$ is the equation of a circle. Identify its center and radius.

Center =

Radius =

#	Points	Score
1	5	
2	6	
3	6	
4	10	
5	8	
6	6	
7	15	
8	4,6	
9	10	
10	10	
11	10	
12	10	
algebra	106	
13	16	
14	10	
15	16	
16	8	
17	8	
18	6	
19	6	
20	6	
21	10	
22	8	
trig	94	
total	200	

6. There is a theorem that expresses the **algebraic formulation** of the method used for the first step to solve all equations like these. **Write that theorem** (that applies to **only the first step**). [Do not find the numbers, state the method, in symbols, for the first step. Do not use English. Make it look like a theorem in a textbook.]

$$(x - 4)^2 = 26 \quad (\cos(x))^2 = .28 \quad (\log x)^2 = 3$$

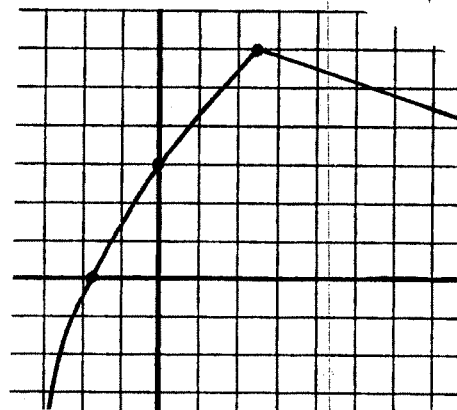
7. Here is a representative graph of $f(x)$. Grid lines are one unit apart.

a) Solve for x : $f(x) = 5$.

b) Solve for x : $f(x) = -x$

c) Sketch, on the same axis system, $y = (1/2)f(x - 3)$.

[Get the 3 marked points right and sketch in the rest.]



8. Short answer. [No work required.]

a) A population undergoes exponential growth with doubling time d . How long will it be until there is eight times the original amount?

b) Rewrite " $42 < x < 68$ " in the form " $|x - c| < d$ ".

9. You know these three facts.

Facts: $f(5.1) = 10.9$, $f(8.2) = 8.8$, and $f(10.1) = 4.9$.

You want to approximate the solution to the equation $f(x) = 6.7$. Use linear interpolation with the appropriate points to do two things: a) Find the most relevant line, and then use it to b) Approximate the solution to $f(x) = 6.7$.

10. You want to rent a car and they give you two choices. On Plan A you can pay \$40 per day plus 25 cents per mile. On Plan B you can pay \$65 per day plus 10 cents per mile. When is Plan B cheaper for you? [Set up the formulas. Set up the relation between them. Then solve.] Answer in a **complete sentence in English**. (The right number without a complete sentence will not get full credit.)

11. Solve algebraically: $50(2^t) = e^{3t}$

12. [This is about the values of currencies.] The euro is worth 37% more than the dollar and the British pound is worth 20% more than the euro. Therefore, the dollar is worth less than the British pound. How much less? [Answer in percents.]

Part II: Trigonometry. Set your calculator to **DEGREE** mode to start. Switch to radian mode when appropriate. For your information: Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.

Law of Sines: $(\sin A)/a = (\sin B)/b$. $\sec \theta = 1/(\cos \theta)$, $\csc \theta = 1/(\sin \theta)$, $\cot \theta = 1/(\tan \theta)$.

13. Short answer:

a) Give θ in degrees, $180^\circ < \theta < 270^\circ$ (in the third quadrant), such that $\sin \theta = -0.6$.

b) Solve $\cot \theta = 0.75$ for θ in degrees in the first quadrant.

c) If $\theta = 700^\circ$, give the reference angle of θ .

d) How many radians are in one degree, **exactly**. [Do not give a decimal answer.]

14. A triangle has sides 3, 15, and 17. What is its area?

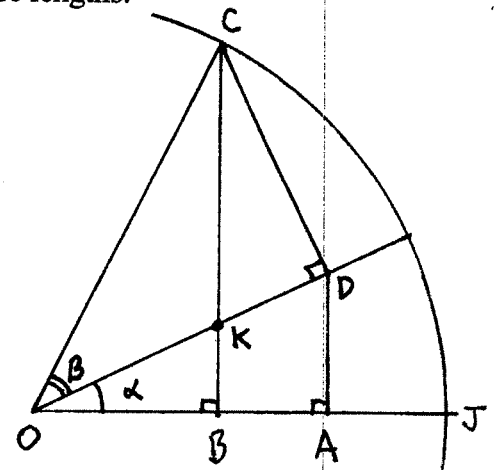
15. The unit-circle figure has angles α and β as in the text and perpendicular lines as in the text. However, K was not in the text's figure. Express simply, in terms of α and β , these lengths:

a) CD

b) OB

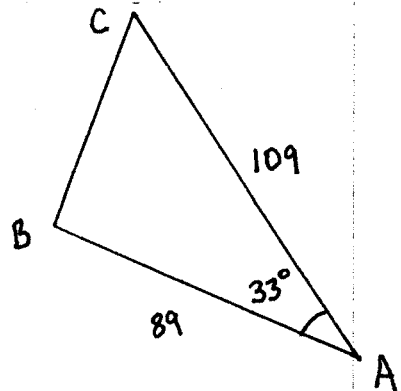
c) DK

d) OA



16. **Derive** the formula for the length of an arc of a circle if the central angle is in degrees. [The formula alone is worth little. Demonstrate to us you know why it is the right formula.]

17. See the figure. $AB = 89$. Angle $A = 33^\circ$. $AC = 109$. Find angle B . [Label the figure with your results as you go along. Tell us what you are doing, but use your calculator programs to do the computations.]



18. Sketch and fully **label** an excellent, and illuminating unit-circle picture to determine and illustrate the usual trig identity for $\cos(-\theta)$. [The identity alone will be worth little. The picture will be marked on how illuminating and how completely **labeled** it is.]

a) **Label**, in the proper locations, at least these:
 θ , $-\theta$ (these two angles must have labeled arcs),
 and $\cos(-\theta)$ (the location of this length must be labeled). [Make θ not near $\pi/4$.]

b) Give the usual trig identity for $\cos(-\theta)$.

19. When the angle is in the first quadrant find $\cos(\tan^{-1}(2x))$. [Show work, of course. Answer in terms of algebraic functions of x , without trig functions.]

20. Here are identities you might wish to use in this problem:

$$(7.3.1A) \quad \sin(\alpha + \beta) = (\sin \alpha)(\cos \beta) + (\cos \alpha)(\sin \beta)$$

$$(7.3.1B) \quad \cos(\alpha + \beta) = (\cos \alpha)(\cos \beta) - (\sin \alpha)(\sin \beta)$$

Derive the identity for $\cos(\alpha - \beta)$ that we derived from one of these. [The point is to show details of the derivation. The correct identity alone is worth little.]

21. Solve algebraically for θ in degrees in the first quadrant. [Show work! You must show clear supporting algebraic work.]

a) $(\cos \theta)^2 = 1.5\sin(\theta) - .3$

b) $\cos(\theta) = 1.7\sin(\theta)$

22. See the figure of a unit circle. If the arc from B to P is twice as long as OA, find the central angle θ . [Set up the equation, tell how to solve it, and give an answer with two significant digits.]

