Precalculus Final, Fall 2014 Name____________________

Section, Instructor, or class time of day____________________

SHOW SUPPORTING WORK!! Little or NO CREDIT will be given unless appropriate supporting work is displayed (except on one-step problems). You must use "algebraic" methods whenever possible. If (and only if) no algebraic method works, guess-and-check is legal and expected. As usual, give answers to at least 3 significant digits.

Problem	Points	Score
1	6	
2	4,4,4,9,4=25	
3	6,6 = 12	
4	10	
5	6	
6	8	
7	5	
8	6	
9	12	
10	10	
algebra	(of 100)	
11	4@ = 24	
12	12	
13	8	
14	8	
15	8	
16	8	
17	16	
18	8	
19	8	
trig	(of 100)	
Total	200	

1. (No work need be exhibited. Answers with fewer than 3 correct significant digits will not get full credit.)
Let $d = -13.9$. Evaluate:

$$\frac{6+\sqrt{d^2+200}}{(d-13)(d+4)}$$

2. Short answer:

a) You look at a graph in the standard window and it goes up from left to right. If you change the y-interval to [- 4, 4] and don't change the x-interval, it will probably appear [pick one]
i) steeper ii) about the same steepness iii) less steep

b) Definition: $s_n = a_1 + a_2 + a_3 + \ldots + a_n$.
If $a_1 = 2$, $a_2 = 4$, $a_3 = 5$, $a_4 = 1$, $a_5 = 3$, and $a_6 = 7$, find s_3.

c) $P(x)$ is a polynomial that goes through these points:
(3, 5), (0, 7), (4, 2), (6, 0).
Find **one** non-constant factor of $P(x)$.

d) Suppose all we know about the graph of $f(x)$ is that $f(3) = 7$.

d1) Find a point on the graph of $f(x - 2)$.

d2) Find a point on the graph of $f(2x)$.

d3) Find a point on the graph of f^{-1}.

e) **State**, in algebraic notation (as in Section 1.4 on reading and writing math), **the method** for evaluating all expressions similar to these in terms of simpler operations: 3 - 5, 17 - 21, 19 - 42.

3. Consider the graph of $y = x^2$ and the points on it where $x = c$ and $x = c+h$.
Find, in point-slope form, the line through those two points. Simplify the slope and give your answer in point-slope form (Do not give slope-intercept form).

4. A point P is on the line $y = 2x$. It is 6 units from (4, 0). Find P.

5. Solve for p (as always, algebraically if possible, and show work!)
$x^p(x^2) = (x^8)\sqrt{x}$, for all x.

6. [SET UP an equation with one variable for solving this. Do not solve it; just set it up.] Spending on healthcare is part of all spending. Suppose spending on healthcare is now 8% of all spending. If spending on healthcare increases at 6% per year and the rest of all spending increases at 2% per year, when will all spending be twice what it is now? SET UP one equation with one unknown that could be solved to answer this.

7. A stock went up 180% over 7 years. What was the average rate of increase per year?

8. One earthquake is 4.2 on the Richter scale. A second is 5.8. The waves of the second have amplitude how many times as much as the waves of the first?

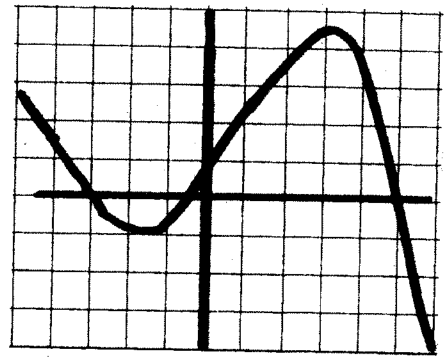
9. Here is a representative graph of f. Grid lines are one unit apart.

a) Find $f(1)$

b) Solve $f(x) = 2$.

c) Solve $f(x) < 1$.

d) Solve $f(2x) = -2$.



10. [Assume exponential decay.] At time zero the amount of a radioactive substance is 0.002 grams. The amount decreases 10% every 25 minutes. Find a formula for the amount remaining after t minutes. [If the formula has parameters that can be determined, determine them.]

Part II: Trigonometry. Set your calculator to DEGREE mode to start. Switch to radian mode when appropriate. For your information: Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.
Law of Sines: $(\sin A)/a = (\sin B)/b$. $\sec\theta = 1/(\cos\theta)$, $\csc\theta = 1/(\sin\theta)$, $\cot\theta = 1/(\tan\theta)$.

11. a) Solve $\sin\theta = -.9$ for θ in degrees in the third quadrant. [Is your calculator in **degree** mode?]

b) Find csc 72°.

c) Solve $\sec\ x = 3.1$ (for x in the first quadrant).

d) Give the reference angle of $19\pi/16$.

e) Exactly how many degrees are in 1 radian (do not give a decimal answer).

f) (In radians) How many solutions does $\sin(x) = .9$ have in the interval $[0, 8\pi)$?

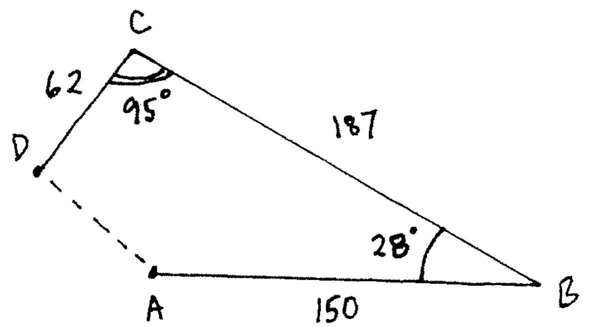
12. [See the picture which has all the given parts labeled] AB = 150, BC = 187, and CD = 62. Angle ABC = 28°. Angle BCD = 95°. Give a PLAN to find AD. Then fill in the correct numbers on the figure.

Correct numbers are not enough. You MUST write enough to demonstrate you know how to do it. Make a clear plan we can easily follow. Label your steps (1), (2), (3), etc. and state what law you are using at each step. Make clear which is your final answer.

(step 1)

(step 2)

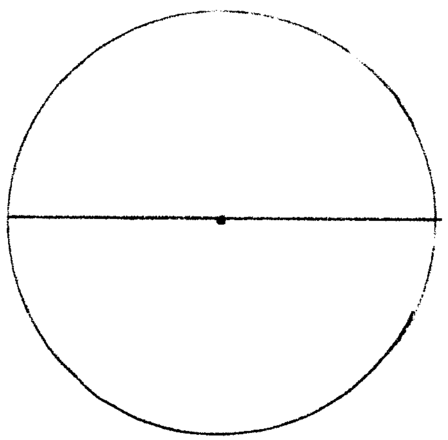
etc.



13. Solve algebraically for θ in the first quadrant: $(\sin\theta)^2 = .8\cos\theta - 0.17$
[Use **degree** mode on your calculator.]

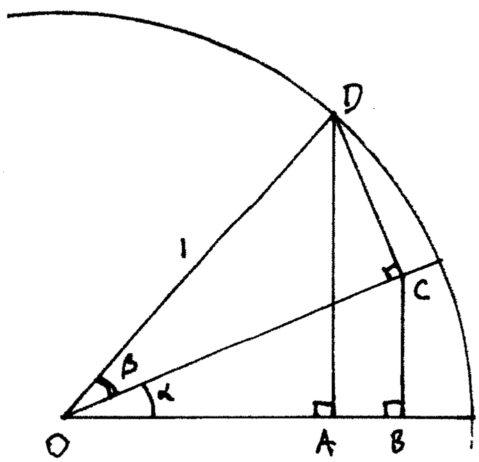
14. Find $\sin(\tan^{-1}(4/x))$ in terms of x when the angle is in the first quadrant.

15. a) Sketch and **fully label** an excellent and illuminating unit-circle picture to determine and illustrate a trig identity for $\sin(\theta - \pi/2)$. Be sure to label the angles θ and $\theta - \pi/2$ and the location of $\sin(\theta - \pi/2)$.
b) Give the usual identity for $\sin(\theta - \pi/2)$. [The identity alone will be worth little. The picture will be marked on how illuminating and how completely labeled it is.]



16. A sector (pie-shaped piece) of a circle has radius r and central angle θ in degrees. Derive the formula for the area of the sector, in terms of r and θ. [The correct answer with no derivation will be worth little. You may assume we know the area of a circle and the circumference of a circle, but do not use more-sophisticated formulas.]

17. In the unit-circle picture, the two angles are α and β, as labeled. [Answer (a)-(c) in terms of α and/or β].



a) Give DC.

b) **Derive** OB

c) **Derive** AB.

d) If $\alpha = 22°$ and AD = 0.73, find β. [Show work, of course.]

18. Here are identities you may wish to use in this problem:

(7.3.3A)	$\sin 2\theta = 2(\sin\theta)(\cos\theta)$
(7.3.3B)	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
(7.3.3C)	$\cos 2\theta = 1 - 2\sin^2\theta$
(7.3.3D)	$\cos 2\theta = 2\cos^2\theta - 1$.

Derive the "half-angle" identity for $\cos(\theta/2)$ that we derived from one of these. [The identity alone is worth nothing. The derivation is everything.]

19. The figure is part of a circle of radius 2. If the line segment AB = 3.09, find the arc length from A to B. [Explain what you did.]