Math 151. The Language of Mathematics   Name _____

Final Exam, on Chapters 1-5.2 Spring 2006.

1. Write out how these should be said aloud so that someone not looking would know what you meant.

a) $(3x)^2$

b) $f(x) = 2(x + 3)$.

2. State the algebraic formulation (symbolically) of the procedure used to evaluate the sum of any two fractions in terms of simpler operations. (Write a method for doing problems like $2/3 + 5/7$.)

3. Here is a sentence: "$3(x + 2) = 3x + 6$."
What is the subject of that sentence? (Be precise.)

4. True or False? If it is true, just say so.
If it is false, also give a counterexample.

a) T F $(a + 2)^2 = a^2 + 4$

b) T F If $b > 0$ and $|c| < |b|$, then $c < b$.

c) T F If $c > 0$ and $|c| < |b|$, then $c < b$.

d) T F If $|x| < 10$, then $x < 15$.

e) T F $bc = 0$ iff $b = 0$ and $c = 0$.

5. Find

a) $(-3, 5) \cap [0, 7]$

b) $(-3, 5) \cup [0, 7]$

c) $(1, 5]^c$

Prob	Points	Score
1	6	
2	6	
3	4	
4	21	
5	9	
6	5	
7	6	
8	20	
9	4	
10	10	
11	10	
12	10	
13	9	
14	10	
15	6	
16	10	
17	6	
18	10	
19	6	
20	12	
21	8	
22	12	
Total	200	

6. Give the (preferably sentence-form) definition of set intersection.

7. Sketch and shade a Venn diagram (two circles inside a rectangle) for $S \cap T^c$.

8. Use the Rules in Section 2.3 to solve these equations. In each case you must do it step-by-step, and for each step exhibit the proper connective, and give the justifying reason. [Do NOT use the Quadratic Formula.]

a) $x^2 + 6 = 5x$

b) $\sqrt{2x + 13} = x - 1$

9. Is the letter x a placeholder? (Yes or No.)

a) $x^2 = c$ b) $(x + 1)^2 = 16$ c) $x^2 = c$ iff $x = \pm \sqrt{c}$

d) Let $f(x) = (x + 1)^2$.

10. Let $f(x) = 5x + 2$. [Use this f for all parts of this problem.]

a) What is the function f that this notation defines?

b) Find simpler $g(x)$ and $h(x)$ such that $f(x) = g(h(x))$.

     $g(x) =$ $h(x) =$

c) Find $f(f(x)) =$

11. Sam invested $10,000 by dividing it between two investments. The value of one investment doubled. The other one went down to half its original amount. The total was then $16,250. How much of his $10,000 was invested in the investment that doubled?

a) Write out all the relevant algebra.   b) Then solve it using algebra.

[Guessing the correct answer is worth little.]

12. Make a complete truth table for "(not A) or B."

13. Here is the form of a conditional "$A \Rightarrow B$."
a) Give its contrapositive.

b) Give its converse.

c) Give its negation.

14. Which of these are logically equivalent to which others of these?
        [Note: We are not asking if they are true.]
Definition: A series which does not converge is said to diverge.
a)   If a series converges, its terms go to 0.

b)   If a series diverges, its terms do not go to 0.

c)   If its terms go to zero, a series converges.

d)   If its terms do not go to 0, a series diverges.

e)   Either its terms go to 0 or a series diverges.

15. Suppose this is false: "For all $x > 5$, $f(x) < 10$." What must be true?

16. a) Define "n is an even number." (Formally, the way it was done in the text.)
b) Use the definition to prove, "If n is even, then n^2 is even."
[Be sure to cite the justification for each step.]

17. True or false? Explain why. "If $x < 10$, then there is y such that $x < y < 10$."

18. Definition: p is an interior point of S iff there is some interval (a, b) such that $p \in (a, b) \subset S$. If the answer is "No," just say so. However, if the answer is "Yes," prove it.

a) Is 3 an interior point of $\{1, 2, 3, 4, 5\}$?

b) Is 3 an interior point of $(0, 3) \cup (3, 5)$?

c) Is 3 an interior point of $[1, 5]$?

19. The <u>Quadratic Theorem</u>: If $a \neq 0$,

the solution to $ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Find the solution for x to this equation [Do not bother to multiply out the terms in the Quadratic Formula.]

$$cx^2 + ax - dx = 3k.$$

20. Definitions: b is an upper bound of S iff if $x \in S$ then $x \leq b$.

b is a bound of S iff $|x| \leq b$ for all x in S.

Theorem: Bounds are upper bounds.

a) Restate the theorem using letters where appropriate.

b) Fill in a justification for each step in the argument. (Four reasons.)

"Argument":   <u>justification</u>

[Step 1] Let b be a bound of S.

[Step 2] Let $x \in S$.

[Step 3] Then $x \leq |x|$

c) Finish with Step 4 and its reason:

d) Why is the proof done?

21. Here is an instruction: Given the expression $x^2 + bx + c$, to complete its square give the expression $(x + b/2)^2 - (b/2)^2 + c$.

   Read it and use it to complete the square of $x^2 + 5x - 4$.

22. Pick ONE of the two options for a substantial essay. Your essay must make at least five relevant points, and not miss any of the most important points.

Option A: Define and explain placeholders.

Option B: Name and explain the ways that two mathematical sentences can appear different yet say the same thing.