Name

The Language of Mathematics Exam 4, on Chapter 4, Spring 2012.

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	tot
pts	8	6	5	6	6	10	8	4	12	3	6	5	6	9	6	100

1. True or false? If it is true, just say so. However, if it is false, also give a counterexample.

a) T F If x + y > 1, then  $x > \frac{1}{2}$  or  $y > \frac{1}{2}$ .

b) T F x > 0 and y > 0 iff xy > 0.

c) T F If x is positive, then |x + y| = x + y.

2. What are the three basic types of sentences with one variable? Give an example of each (and be sure you are clear about which example goes with which type).

3. In which of these is x or T a placeholder? a) Let f(x) = 3(x+1). b)  $x(x+2) = x^2 + 2x$  c) If x > |c| then x > c. d)  $S \cap T \subset T$  e)  $\emptyset \subset T$ 

4. Identify the types of all the variables in the sentence. (Some variables have more than one associated term).
a) x - a = b.

b)  $x^2 = c$  iff  $x = \pm \sqrt{c}$ .

5. Rewrite the sentence using "If ..., then ...". a) f(x) > 9 whenever x > 3

b) For all x in S,  $|x| \leq 12$ .

c) Numbers ending in 3 have squares ending in 9.

6. Here are two sentences with some similarities but important differences. Note the important differences between them by contrasting them with a comment about the first in the left column and a contrasting comment about the second in the right column. [Make at least 5 contrasts.]

 $3(x+5) = 3x+15 \qquad \qquad 3(x+5) = 15$ 

7. What is this sentence about? (Do not write out its pronunciation or name it or repeat what it says-identify the thing that is the subject of the sentence.) a)  $x^2 \ge 0$ 

- b) x + 1 > x
- c) Let  $f(x) = x^2$
- d) If  $S \subset T$  and  $T \subset R$ , then  $S \subset R$ .
- 8. Here is a definition: Let  $x \Box y = 2y x$ . a) Find 7  $\Box$  5
- b) Express  $5 \Box x$

9. Give the **negations** of these sentences (in positive form):

a) If  $x \in S$ , then x < 10.

b) All boxes in the van weigh at least 40 pounds.

c) There is a zap with no wyps.

d) All values of f(x) are less than 10.

10. Here is a sentence: "If x > 7, then  $f(x) \ge 10$ ." Do these facts prove it is false? (Answer "yes" or "no".) a) Fact: f(4) = 9

b) Fact: f(11) = 12

c) Fact: f(9) = 8

11. Definition: *b* is an upper bound of *S* iff if  $x \in S$  then  $x \leq b$ . a) Give the contrapositive of "15 is an upper bound of *T*." (Translate first, then give the contrapositive of the translated version).

b) Give the negation of "15 is an upper bound of *T*." (Translate first, then negate the translated version. Make your answer in positive form).

12. True or false? Just circle one.

- a) T F Upper bounds are never negative.
- b) T F There exists x > 5 such that  $x^2 < 26$
- c) T F If x is in the empty set, then x < -12.
- d) T F For all  $x \ge 0$  there exists  $y \ge 0$  such that  $y \le x$ .
- e) T F There exists  $y \ge 0$  such that for all  $x \ge 0$ ,  $y \le x$ .

13. Give a pair of illuminating examples to distinguish between "all are not" and "not all are." Explain the distinction.

14. The <u>Quadratic Theorem</u>: If  $a \neq 0$ , the solution to  $ax^2 + bx + c = 0$  is given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

Use it to give the solutions to these

[Do not bother to multiply out or simplify the terms in the Quadratic Formula.]

- a)  $5x^2 + 4x 29 = 0$ .
- b)  $cx^2 + 2kx + = 12$
- c) Solve for  $c: 2c + kc^2 = d$ .

15. Here is a theorem. Read it and use it to do the problem [Do not bother to multiply out your answer.]

Theorem: The derivative of  $c(\cos(kx))$  is  $-ck(\sin(kx))$ 

- a) Find the derivative of  $5\cos(2x)$
- b) Find the derivative of cos(x/3)