Math 151. The Language of Mathematics Name ______________________

Exam 4, on Chapter 4. Spring 2006.

#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	tot
pts	5	4	4	6	6	9	6	10	4	4	12	4	9	9	8	100

1. In which of these is x a placeholder?

a) $|x| \geq 0$ b) $|x| < |x+1|$ c) $x \leq |x|$

d) $|x| < c$ e) $|x| < c$ iff $-c < x < c$

2. Do the two sentences have the same meaning? (Just answer "Yes" or "No")

a) $x^2 = 9$ $y^2 = 9$

b) $f(x) = x^2$ $g(x) = x^2$

c) $f(x) = x^2$ $f(z) = z^2$

d) $(x + 2)^2 = x^2 + 4x + 4$ $(y + 2)^2 = y^2 + 4y + 4$

3. Explain (clearly and technically) the distinction between a sentence with a variable and a statement.

4. Give the negations of these sentences (in positive form):

a) If $x \leq 8$, then $f(x) > 10$.

b) There is a box in that stack with more than 20 books.

5. In each part of this problem determine which of these statements apply to the row.
#1) all are 3's; #2) all are not 3's; #3) not all are 3's; #4) there exists a 3.
[Each part requires four decisions. List the ones that apply.]

the ones that apply are:

a) 23888888999

b) 33333333

c) 44448966555

6. True or false. If it is true, just say so. However, if it is false, also give a counterexample.

a) T F $|x + 2| - 2 = x$

b) T F $x^2 = c^2$ iff $x = c$.

c) T F $|b| < |c| \rightarrow b < c$

7. True or false? Just circle one.
a) T F For all $x > 0$ there exists $y > 0$ such that $y < x$.
b) T F There exists $y > 0$ such that for all $x > 0$, $y < x$.
c) T F For all $x > 0$ there exists $y > 0$ such that $y > x$.

8. Definitions: b is an upper bound of S iff if $x \in S$ then $x \leq b$.
b is a bound of S iff $|x| \leq b$ for all x in S.
S is bounded above iff there exists an upper bound of S.
S is bounded iff there exists a bound of S.

a) Is 14 an upper bound of $[-30, 5]$?

b) Is 14 a bound of $[-30, 5]$?

c) True or false? If 100 is an upper bound of T, then 100 is a bound of T.

d) True or false? If 100 is a bound of T, then 100 is an upper bound of T.

e) True or false? If S^c is not bounded, then S is bounded.

9. Define $x\#y = 2x - 5y$ if x is odd and $x\#y = 3x - 2y$ is x is even.

a) Find $2\#4 =$

b) Find $3\#x =$

10. Give a pair of illuminating examples to distinguish between "all are not" and "not all are." Explain the distinction.

11. The <u>Quadratic Theorem</u>: If $a \neq 0$,

the solution to $ax^2 + bx + c = 0$ is given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Give the solutions to these [Do not bother to multiply them out the terms in the Quadratic Formula.]

a) $3x^2 - 6x - 30 = 0$.

b) $2x(x - 3) = 50$.

c) $bx^2 + cx + 2k = 0$

d) Solve for y: $3x^2 + 5xy + 4y^2 = 100$.

12. a) Give the form: "If n is an integer, then 3 divides n or 1 is the remainder when 3 divides n^2."

b) Give a sentence logically equivalent to it, using a logical equivalence from Chapter 3.

13. Here is a theorem. Read it and use it to do the problem [Do not bother to multiply out your answer.]
Theorem: $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6$.

a) Find the sum: $1^2 + 2^2 + \ldots + 60^2$

b) Find the sum: $1^2 + 2^2 + 3^2 + \ldots + (2j - 1)^2 + (2j)^2$

c) Find the sum: $50^2 + 51^2 + 52^2 + \ldots + 200^2$.

14. There are many ways that a sentence can look different but say the same thing. Select three ways mentioned in Section 4.7 on this topic, **name them** and **use them** to restate the given sentence:

$S \subset T$	name of way	alternative sentence that says the same thing as "$S \subset T$"
1)		
2)		
3)		

15. Find which of these are logically equivalent to which others of these. Find all logically equivalent pairs.

a) If $x = z$, then $f(x) = f(z)$

b) If $x \neq z$, then $f(x) \neq f(z)$

c) If $f(x) = f(z)$, then $x = z$

d) $f(x) = f(z)$ or $x \neq z$

e) $x = z$ and $f(x) \neq f(z)$