The Language of Mathematics Name _____

Exam 3, on Chapter 3. Spring 2006.

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pts	10	5	9	5	6	4	6	5	13	4	3	12	10	8	100

1. Fill out the seven columns in the truth table:

A	B	not A	A and B	A or B	A ⇒ B	A iff B

2. a) Define "converse." [Use a complete sentence or sentences.]

b) Give an example of a sentence which is true but its converse is false.

3. We have logical equivalences for several important forms. Give the form we studied that is logically equivalent to

a) not(A and B)

b) not(A ⇒ B)

c) A or B

4. a) Is the solution to "$x^2 = 9$" given by "$x = 3$ and $x = -3$" or is it given by "$x = 3$ or $x = -3$"?

b) **Explain** why the other one is not right. [Use complete sentences.]

5. True or false? [No reason required]

a) T F "$S \subset T$" and "If $x \in S$, then $x \in T$" are logically equivalent.

b) T F "$3x = 15$" and "$x = 5$" are logically equivalent.

c) T F $x < -10 \Rightarrow |x| > 5$.

6. There is a perfect parallel between set-theory terms and logical connectives. Which of the five connectives is used in the definition of the set-theory term?

a) subset b) intersection

7. Give the contrapositive of

a) "If a series converges, its terms go to zero."

b) "Squares have four equal sides."

c) "If not B, then A."

8. Some of the named logical equivalences we studied had three basic components. Name one logical equivalence (that we had as a theorem) with three components and state it (Your choice.)

9. Five sentences follow. The form of the first is given. Using the same meaning for "A" and "B", give the form of each of the others.

	sentence	form
a)	$x < 2$ or $f(x) \geq 5$	A or B

b) If $f(x) \geq 5$, then $x < 2$.

c) If $x \geq 2$, then $f(x) \geq 5$.

d) If $x \geq 2$, then $f(x) < 5$.

e) If $f(x) < 5$, then $x \geq 2$.

Now that you have all five forms identified, find all logically equivalent pairs among these five.

10. Theorem 3.3.9 says: D is logically equivalent to E iff "D iff E" is a tautology.
Theorem 3.1.10 says: "H ⇒ C" is logically equivalent to "(not H) or C."
Give the tautology that corresponds to Theorem 3.1.10.

11. State the simplest tautology.

12. Resolve this conjecture with a truth table with 6 columns (one for each component and connective). "*A* is logically equivalent to (not *A*) ⇒ (*B* and (not *B*))." After you finish the truth table, say if it is true or false, and why.

13. Suppose this is true: "Internists with over 5 years experience can apply to be board-certified." What can be deduced in the following situations? Include conditional answers, too.

a) She is an internist who cannot apply to be board-certified.

b) She can apply to be board-certified.

c) She is an internist.

d) She cannot apply to be board-certified.

e) She has over 5 years experience and can apply to be board-certified.

14. (Medium length essay.) Everyone who studies logic wonders why the truth table for "$A \Rightarrow B$" is defined the way it is. We gave two different types of arguments to explain it. Explain why "$A \Rightarrow B$" is true when it is and false when it is, as if to a student who did not know. [Your essay will be graded on how illuminating and convincing your explanation is, not merely on being "right." You must use complete sentences and clear examples, at least one of which is mathematical.]